POLYOMINO LESSONS

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POLYOMINO LESSONS

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INTRODUCTION

Polyominoes are the shapes made by joining squares edge-to-edge. They were named and first studied by mathematician Solomon W. Golomb, starting in 1953. Martin Gardner's "Mathematical Games" column in *Scientific American* popularized many polyomino puzzles and problems. Pentominoes, the special case of polyominoes of area 5, have enjoyed the greatest success among recreational mathematicians, game players, and puzzle buffs, and are now finding their way into the classroom.

This book provides an introduction to polyominoes for students in grades 4 through 8. Most of the lessons can also be adapted for use in primary and high school classes also.

THE AUTHOR

HOW TO USE THIS BOOK

The only materials required are grid paper and pencils for your students. (Quarter-inch grid paper works well.) You may also use the duplicating masters at the back of this book. A permanent grid on a chalkboard and/or on grid transparencies for the overhead projector can also be helpful in conducting whole-class lessons and demonstrations.

Optional materials: interlocking cubes, Perceptual Puzzle Blocks™, plastic pentominoes, pattern blocks, tangrams, Supertangrams™, geoboards, Soma Cubes.

Allow students to work in groups and/or to share the results of individual work with each other. You may wish to use a bulletin board to post solutions to particularly challenging problems.

A Hints section is provided for some of the more difficult problems. Every problem or question in the activity pages that has a corresponding hint in the Hints section is marked with ★.

If students have their own copies of this book, you will probably want to remove the solution pages. You may also wish to remove the Hints section and share the hints with the students when you feel it is appropriate.

Page-by-page suggestions for using the activities can be found in the Comments for the Teacher section at the end of this book.

Note that there is more than one solution (sometimes, even hundreds) for most of the problems and puzzles in this book. One solution, however, has been provided in the Solution Section for most of the problems and puzzles. Your students will find their own collection of solutions, which will probably be different from those of other groups of students.

This book can serve as an introduction to more work with pentominoes or it can be used concurrently with the various pentomino books

It is essential that your students keep track of their work and their results. They will draw answers to puzzles in the spaces provided on each page or on grid paper. Provide each student with a folder for his or her work.

The following material available from your local library may be of interest:

Gardner, Martin. "Mathematical Games." *Scientific American*, December 1957, November 1960, October 1965, September 1972, August 1975, April 1979. These are the articles that gave polyominoes the widest possible audience among recreational mathematicians and other puzzle buffs.

Golomb, Solomon W. *Polyominoes*. New York: Charles Scribner's sons, 1965. Written by the man who got it all going.

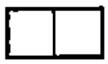
Journal of Recreational Mathematics. Many issues of this journal feature articles, problems, and puzzles on pentominoes and related topics. An indispensable source for polyomino fanatics. See Vol 16 (1983-84) no. 4, page 273, for a bibliography by William L. Schaaf.

FINDING THE POLYOMINOES

Polyominoes

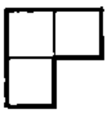
Fill in the blanks in some of the statements. Make some drawings on the grids.

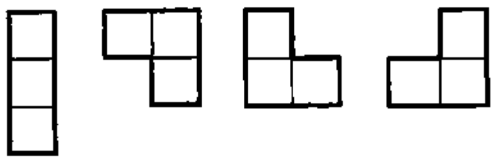
1. This is a domino. It is made up of two squares.



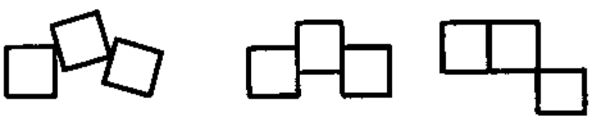
2. This is the straight triomino. It is made up of________squares.

3. This is the bent triomino. There are only 2 triominoes. The following are the same ones, but in different positions.





4. These are not triominoes. The squares are not joined edge-to-edge.



5. A tetromino is made up of 4 squares joined edge-to-edge. Draw one.

6. A pentomino is made up of________ squares joined edge-to-edge. Draw one.

7. A hexomino is made up of ________ squares joined edge-to-edge. Draw one.

All of these shapes are called polyominoes. A polyomino is any figure made up of squares that are joined edge-to-edge.

Tetrominoes

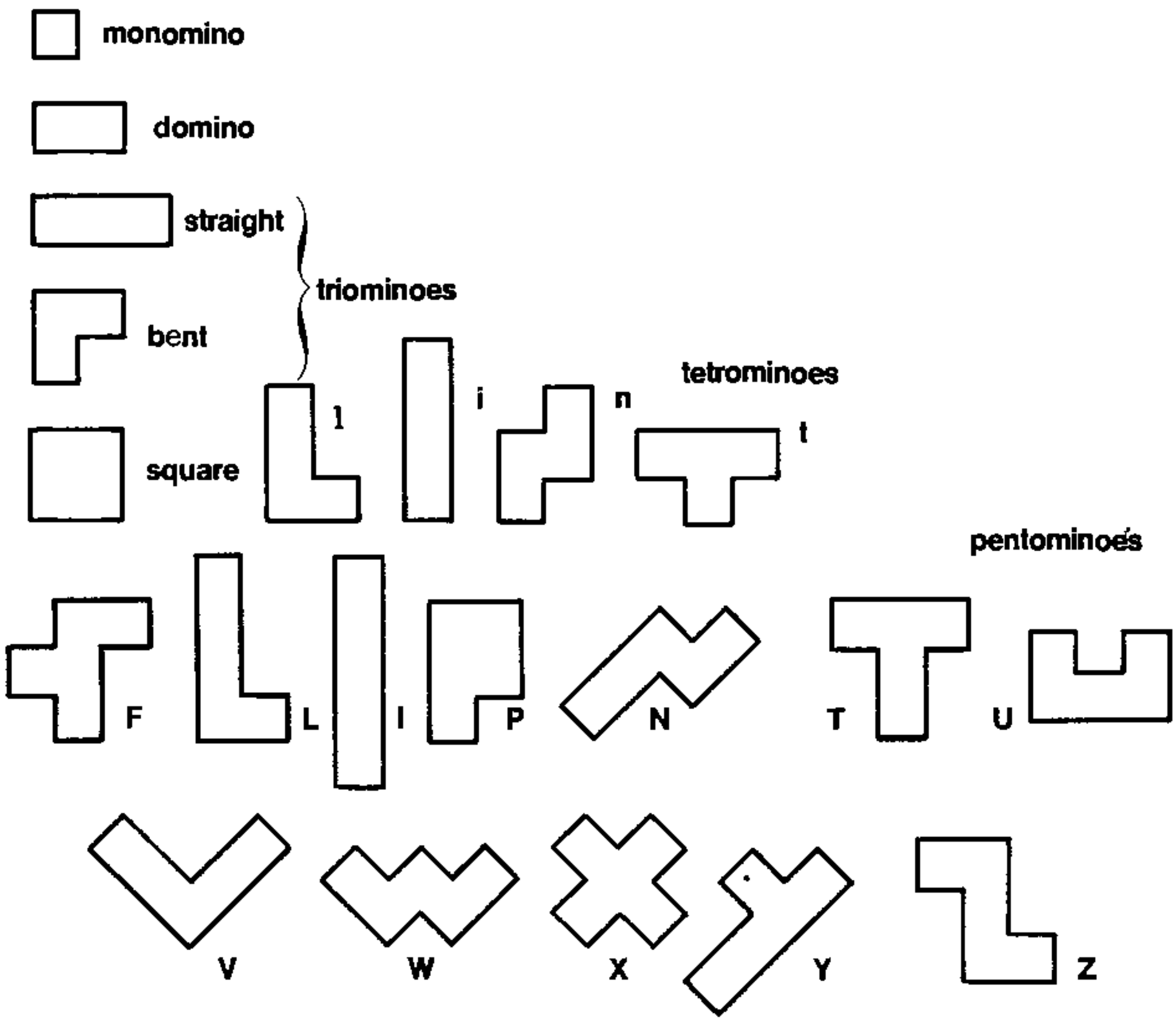
Use this grid to find and draw all the tetrominoes. Make sure you do not show the same one twice. ★

Pentominoes

Use this grid to find and draw all the pentominoes. Make sure you do not show the same one twice. ★

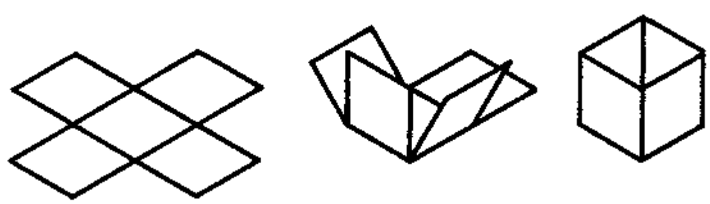
Polyomino Names

People often give names to the polyominoes. The names that will be used in this book are shown in the figure below. They are easy to remember, and you should learn them. Note that the lower case letters refer to the tetrominoes, while the capital letters refer to the pentominoes.



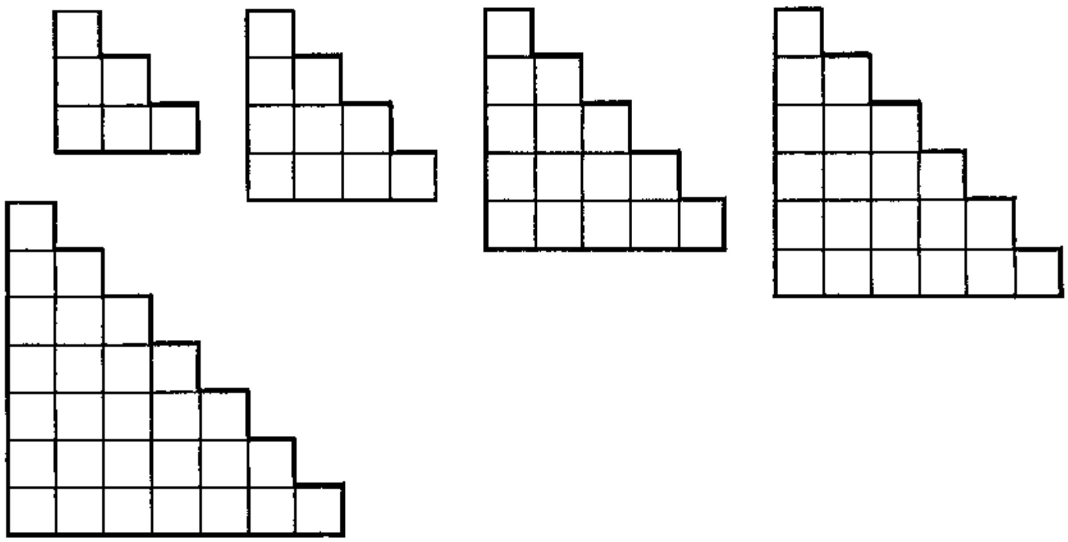
If you cut out paper pentominoes, which ones can be folded into a box without a top? ★ ____________________

Example: folding the X

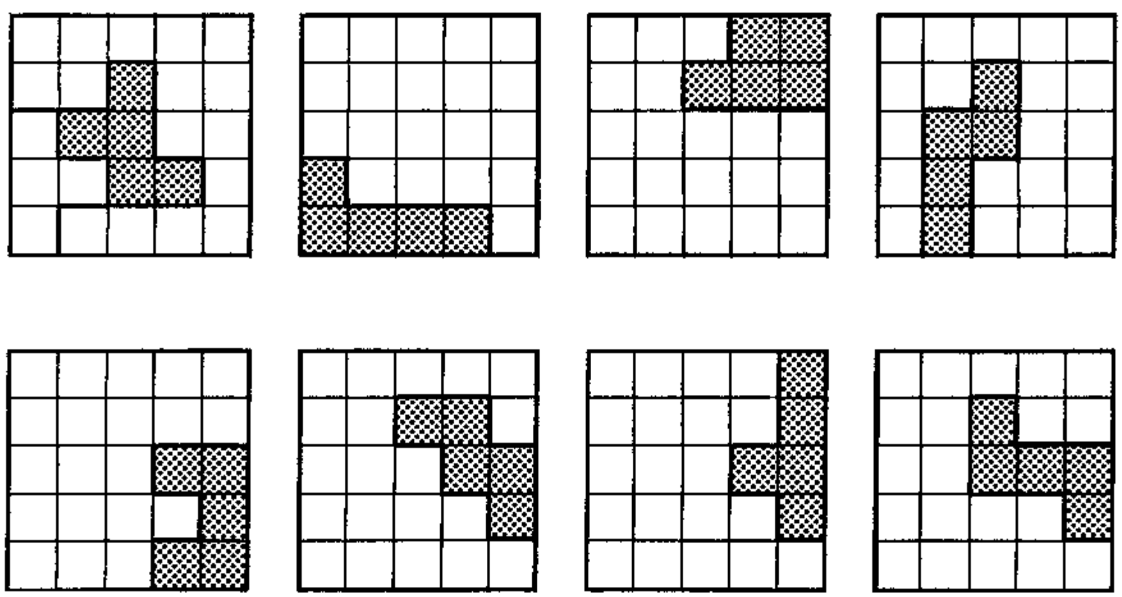


Some Polyomino Puzzles

1. Use the domino, triominoes, and tetrominoes to cover the figures below. Imagine how different shapes might fit. Then draw them on the figures. Never use the same polyomino more than once in the same figure.

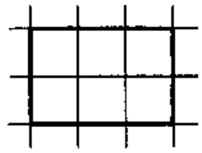


2. Each of the 5 x 5 squares below contains a pentomino. It is shaded. In each square, divide the remaining area into the five distinct tetrominoes. The first one has been started for you.



Making Polyomino Rectangles

This is a 2 by 3 rectangle. Its dimensions are 2 and 3. (That is, its sides are 2 units by 3 units.) Its area is 6 square units.



1. On the grid below draw all the rectangles, including squares, with area 28 or less. Their dimensions should be whole numbers greater than 1. Use more grid paper if you need it. ★

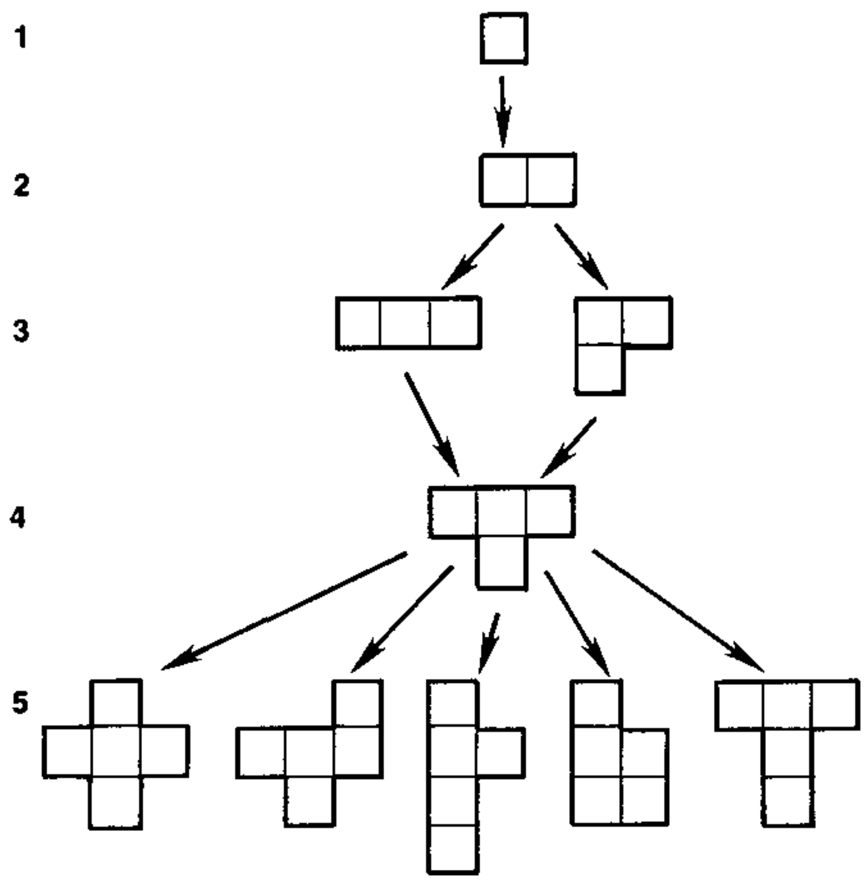
2. Now use the domino, triominoes, and tetrominoes to cover these rectangles. Don't use the same polyomino more than once in the same figure. You will need to use a single monomino in one of the figures, but only one. ★

FAMILY TREES

A polyomino is a child of another polyomino if it can be made from the original polyomino by the addition of a single square. For example, the l, i, and t tetrominoes are children of the straight triomino. The square and n tetrominoes are not.

Here is a family tree for the t tetromino. It shows all its ancestors back to the monomino and all of its pentomino children.

Generation:



Make a complete family tree for the l tetromino. Use grid paper for your drawings.

Tetromino Trees

Make complete family trees for the square, i, and n tetrominoes.

Pentomino Family Relationships

Fill in the blanks on this page.

1. Which pentomino has the most (tetromino) parents? ★ ______

Two polyominoes "of the same generation" are called siblings (brothers and sisters), if they have a parent in common. For example, the F and the P pentominoes are siblings. Both are children of the t tetromino.

2. List all the siblings of the l pentomino. ★ ______

3. List all the siblings of the W pentomino. ______

Polyominoes of the same generation are cousins if they are *not* siblings. For example, the square and the straight tetromino are cousins. They do not have a parent in common.

4. List all the cousins of the Y pentomino. ______

5. Find two "second cousin" pentominoes. These are pentominoes that have no tetromino or triomino ancestors in common. ______

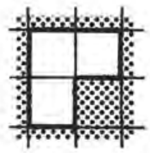
Two extra-challenging problems:

6. Which pentomino has the most (hexomino) children? ______

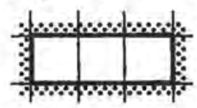
7. Which pentomino has the fewest (hexomino) children? ______

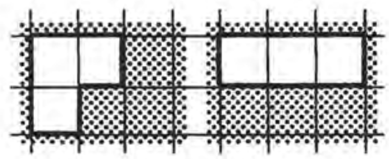
Envelopes

What is the smallest rectangle (or square) onto which you can fit the bent triomino? As you can see in this drawing, it is a 2-by-2 square.



A 1-by-3 rectangle is the smallest rectangle onto which you can fit the straight triomino. The 1-by-3 and 2-by-2 rectangles are triomino *envelopes* because they are the smallest rectangles into which the triominoes will fit. The 2-by-3 rectangles are not triomino envelopes because they are too big.





Find all the tetromino envelopes. Write their dimensions in the blanks below. Also write the names of the tetrominoes that belong in each envelope. The work has been started for you.

1. __1__ by _____ Tetrominoes: __i__________
2. _____ by _____ Tetrominoes: ____________
3. _____ by _____ Tetrominoes: ____________

Pentomino Envelopes

Find all the pentomino envelopes. Write their dimensions in the blanks below. Also write the names of the pentominoes that belong in each envelope.

1. ________ by ________ Pentominoes: ________________________________
2. ________ by ________ Pentominoes: ________________________________
3. ________ by ________ Pentominoes: ________________________________
4. ________ by ________ Pentominoes: ________________________________

Hexominoes

Use this grid to find as many hexominoes as you can. Make sure you do not show the same one twice.

How many hexominoes do you think there are? ________
You probably found a great many. Are you sure you found them all? Are you sure you have no duplicates?

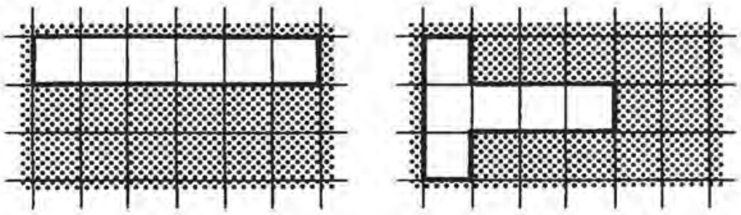
Hexomino Envelopes

Here is a list of rectangles.

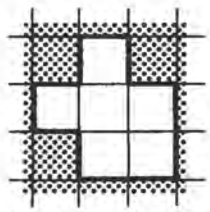
2 by 2	1 by 5	2 by 3	2 by 5
3 by 3	3 by 5	2 by 4	1 by 6
3 by 4	2 by 6	4 by 4	3 by 6

Some of these rectangles are too small to be hexomino envelopes. The 2-by-2 rectangle is made of only 4 small squares and is not one. Cross out the rectangles that are too small to be hexomino envelopes.

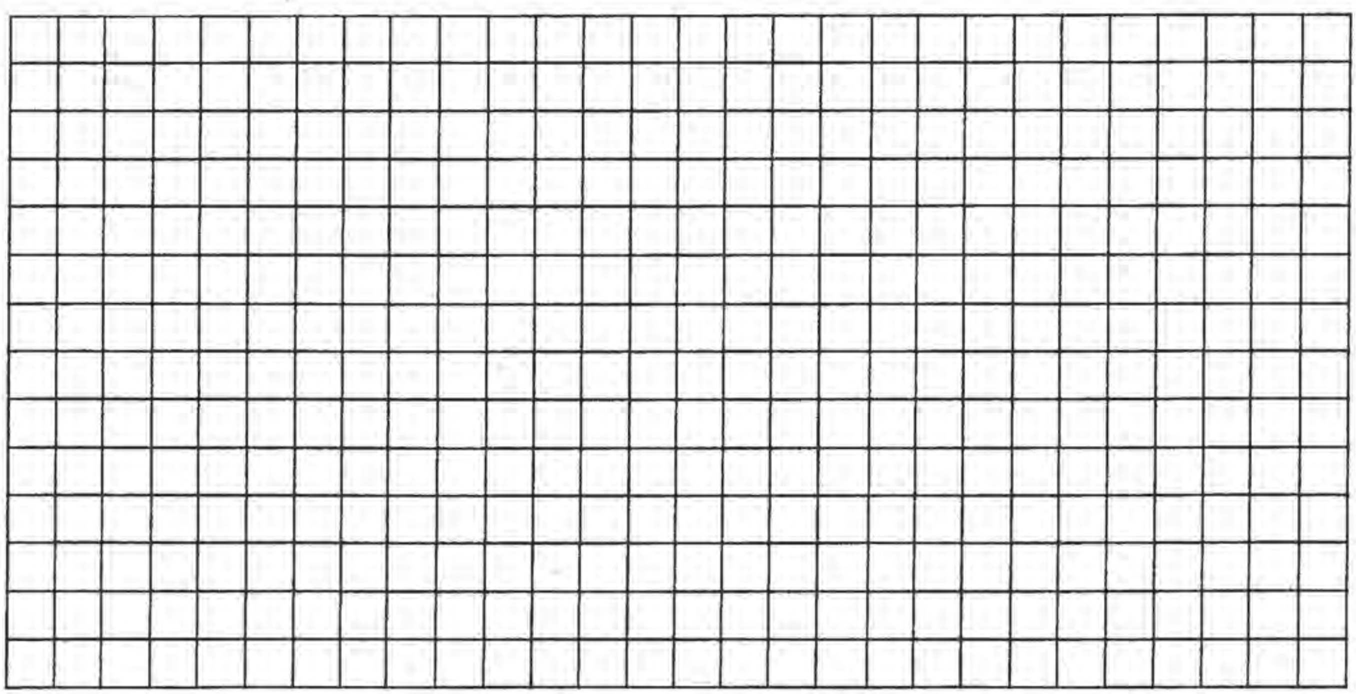
Some of the rectangles are too big to be hexomino envelopes. For example, the 3-by-6 rectangle is too big.



However, the 3-by-3 rectangle is a hexomino envelope.

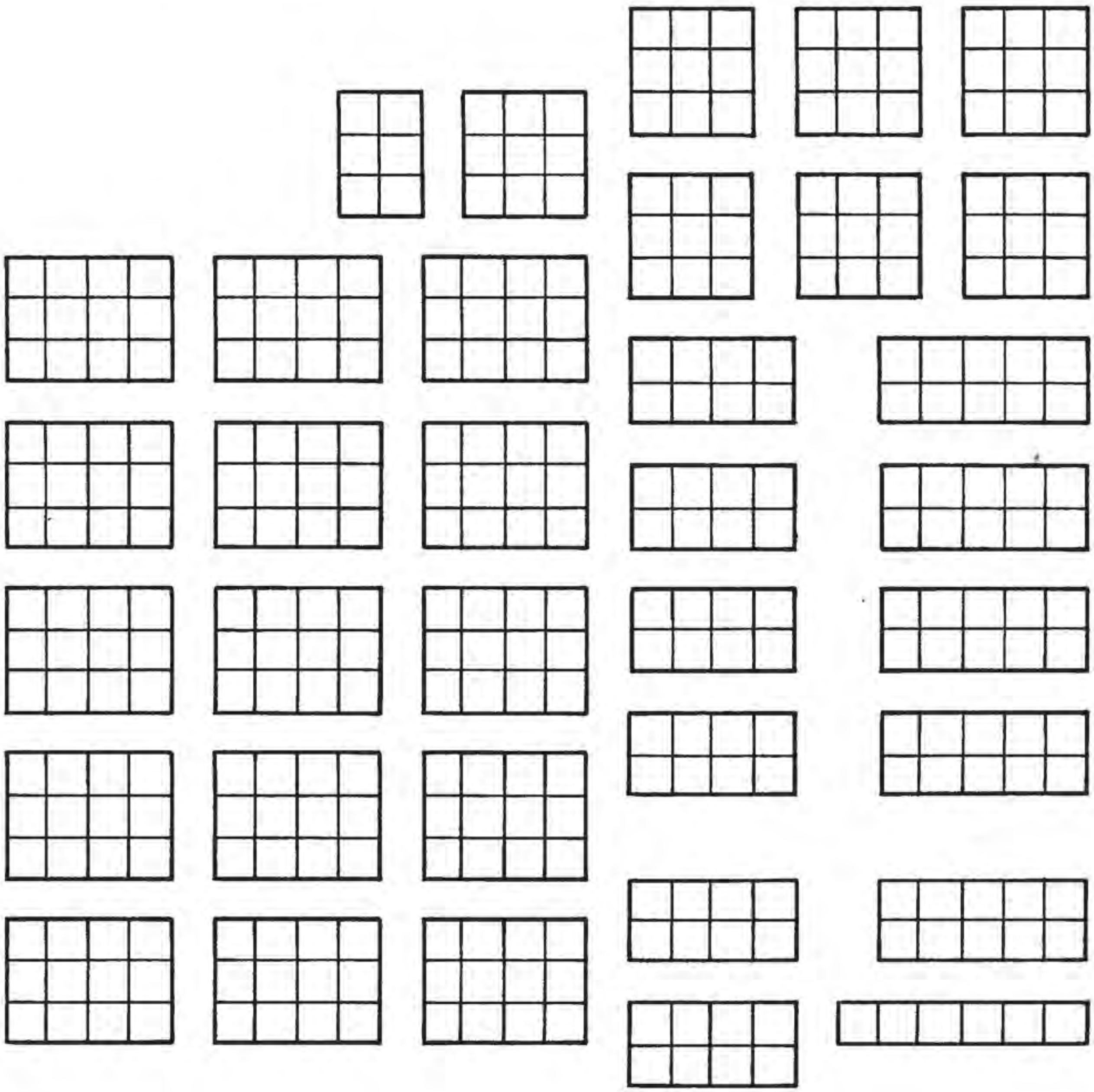


Circle all the hexomino envelopes on the list above. Cross out all the rest. Use the grid below to experiment. ★



Classifying the Hexominoes

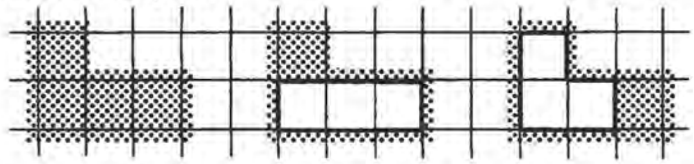
There are 35 hexominoes. Is that the number you found? Organize them by drawing them in the envelopes below. As you work, watch out for duplicates. There should be no empty envelopes when you finish.



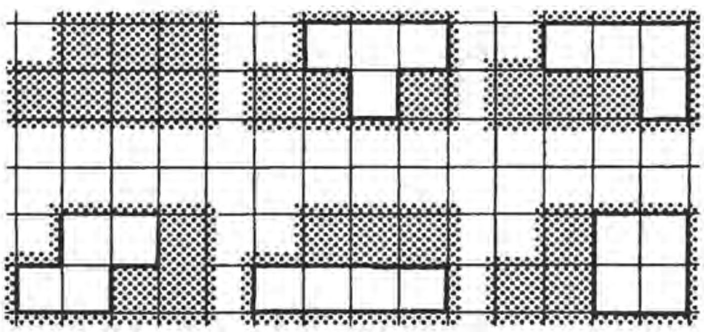
If you cut out paper hexominoes, which ones could you fold into a cube? Mark them with checks above. ★

Minimum Covers

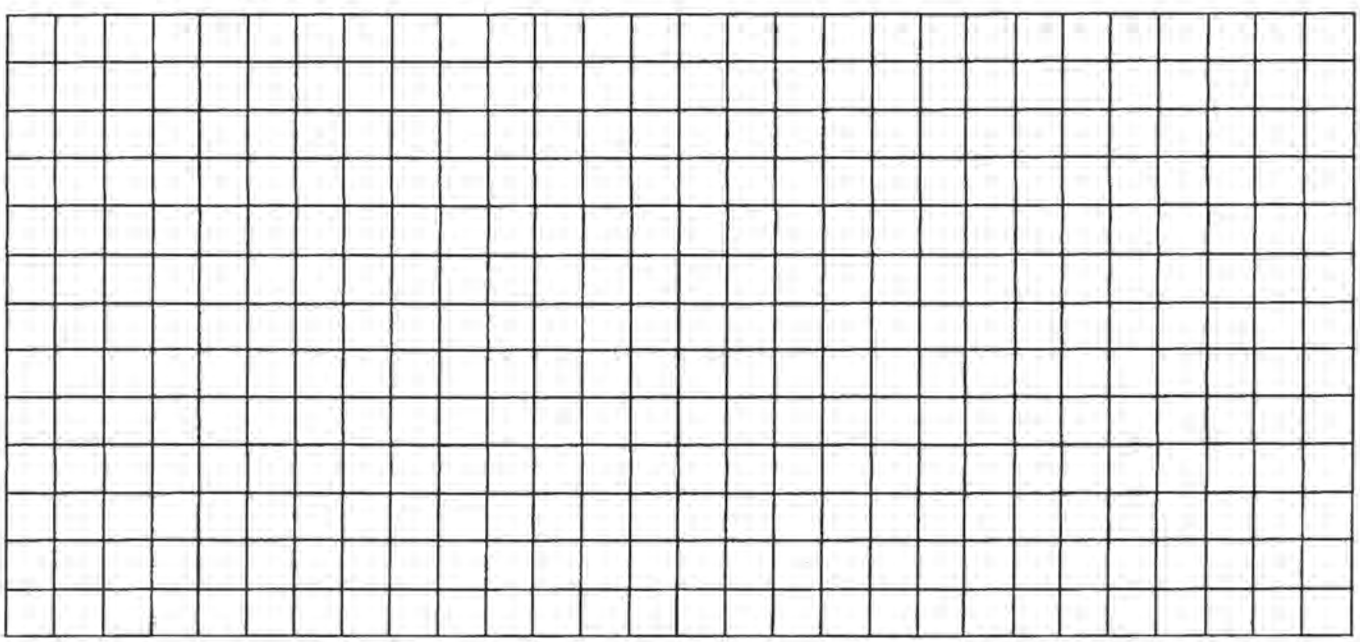
This is the smallest space onto which you can fit either triomino. Its area is 4 square units.



All the tetrominoes fit on this space. Area: 7 square units



1. Find a smaller area on which any of the tetrominoes will fit. Experiment on the grid below. Area:________
2. What is the smallest area onto which you can fit any pentomino? Experiment below. Area:________
3. What is the smallest area onto which you can fit any hexomino? Experiment below. Area:________

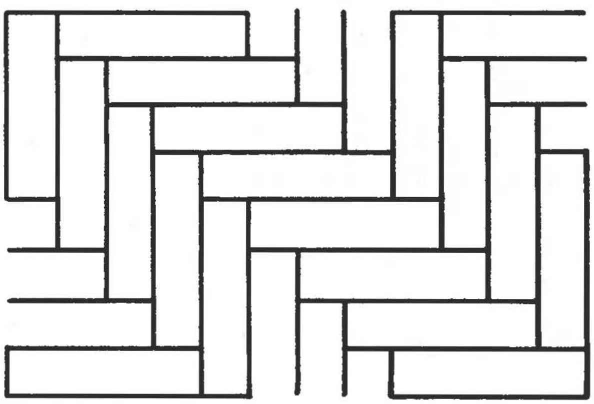


Tiling the Plane

Imagine a flat surface that extends forever in all directions. Such a surface is called a *plane*. If you had an unlimited supply of rectangular polyomino tiles, you could cover as large an area as you wanted. You could lay them out like this.

straight triomino tiling

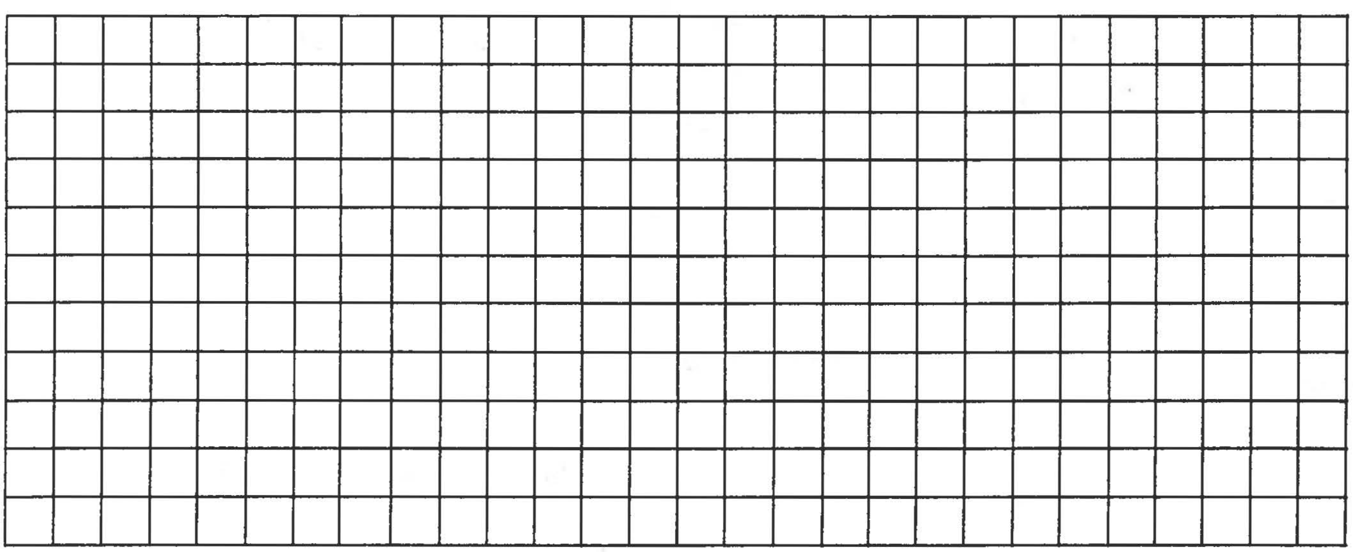
This pattern could be extended in all directions. Other patterns are possible with rectangles. Here is one below:



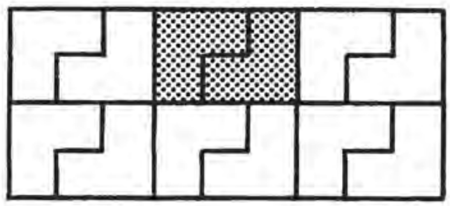
i tetromino tiling

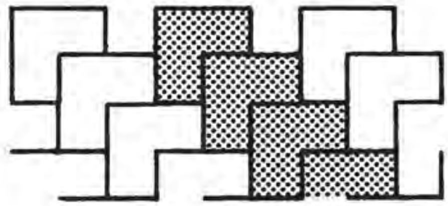
Find interesting ways to tile a plane with the following four rectangular polyominoes. Use the grid below and some grid paper.

1. domino 2. square tetromino 3. I pentomino 4. 2-by-3 hexomino

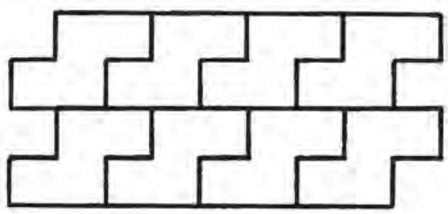


Tiling with Other Polyominoes



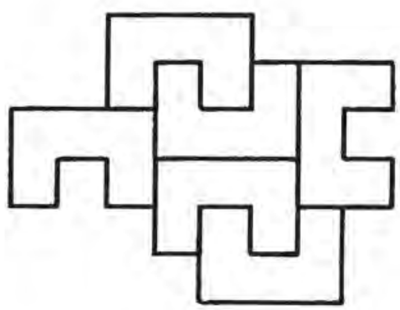


Here are 2 bent triomino tilings. They can be extended in all directions. The first one is made up of two-triomino rectangles. They can be used to tile the plane. The second one is made up of diagonal stripes. By making more stripes beside these, you can continue the pattern as far as you want.



1. Can you continue this n tetromino tiling in all directions? ____________________

   Use grid paper to show if you can do it.



2. Can you continue this U pentomino tiling in all directions? ____________________

   Use grid paper. Tell what happens. ____________________

   ____

Tiling

Show one or more ways to tile with each of these polyominoes.

1. l tetromino
2. L pentomino
3. P pentomino
4. t tetromino

Tiling with Pentominoes

Show one or more ways to tile with each of these pentominoes.

1. Y   2. F   3. N

Tiling with Pentominoes

For each pentomino, show one or more ways to tile.

1. T   2. U   3. V

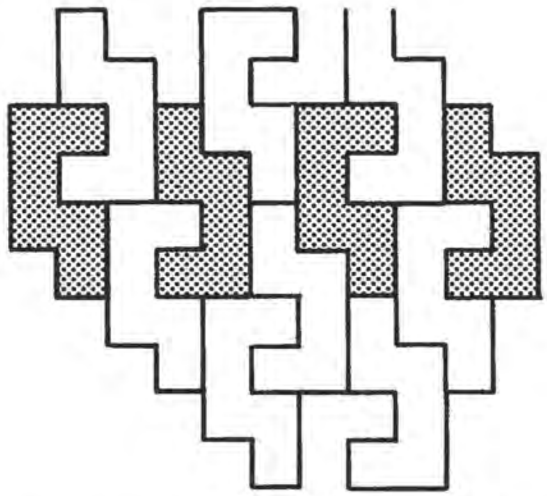
More Tiling with Pentominoes

For each pentomino, show one or more ways to tile.

1. W
2. X
3. Z

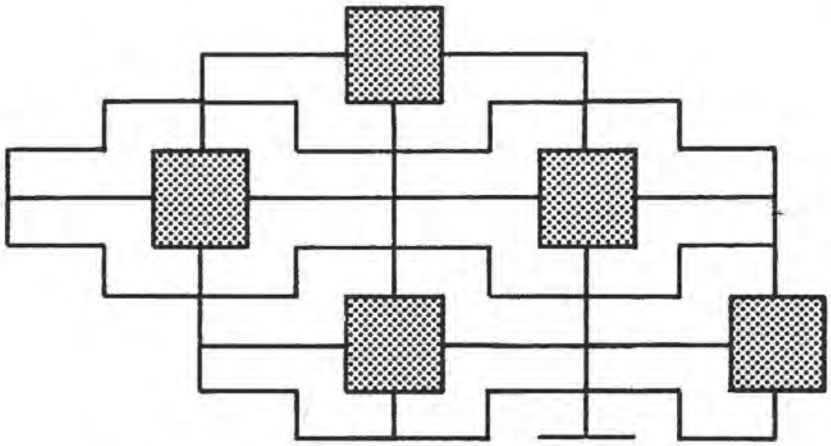
Hexomino Tiling

Here is a hexomino tiling. Make up some of your own.



Tiling with Two Polyominoes

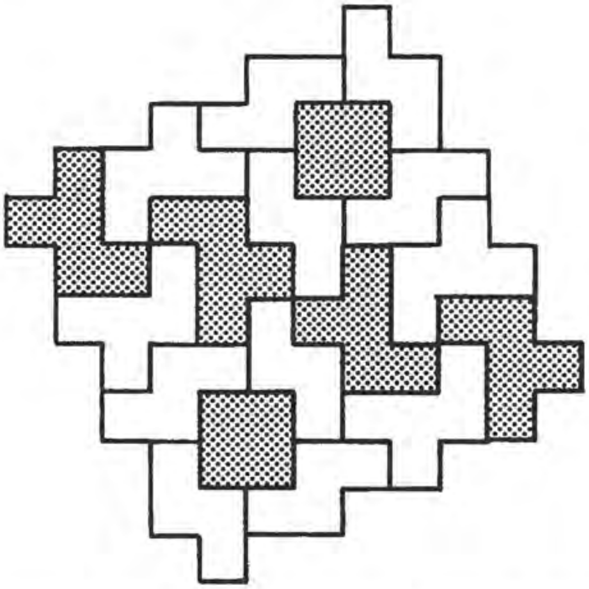
Here is a tiling that uses two polyominoes. Make up some more tilings of your own.



N and square

Tiling with Three Polyominoes

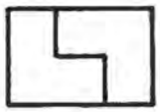
Here is a tiling that uses three polyominoes. Make up some more tilings of your own.



F, n, and square

Tiling Rectangles

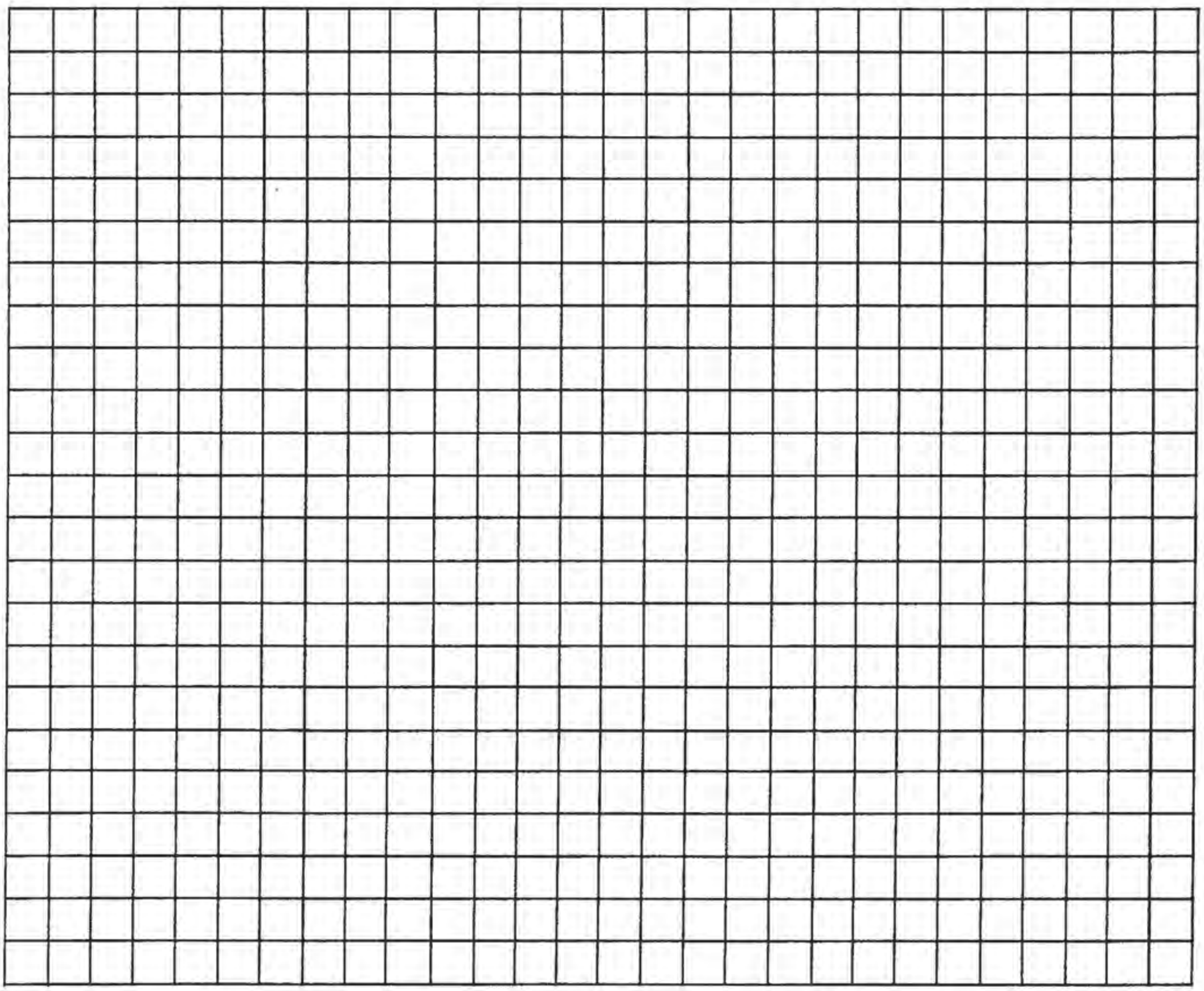
If a rectangle can be covered exactly by copies of the same polyomino, then that polyomino tiles the rectangle. For example, here is the smallest rectangle that can be tiled by the bent triomino.



2 by 3

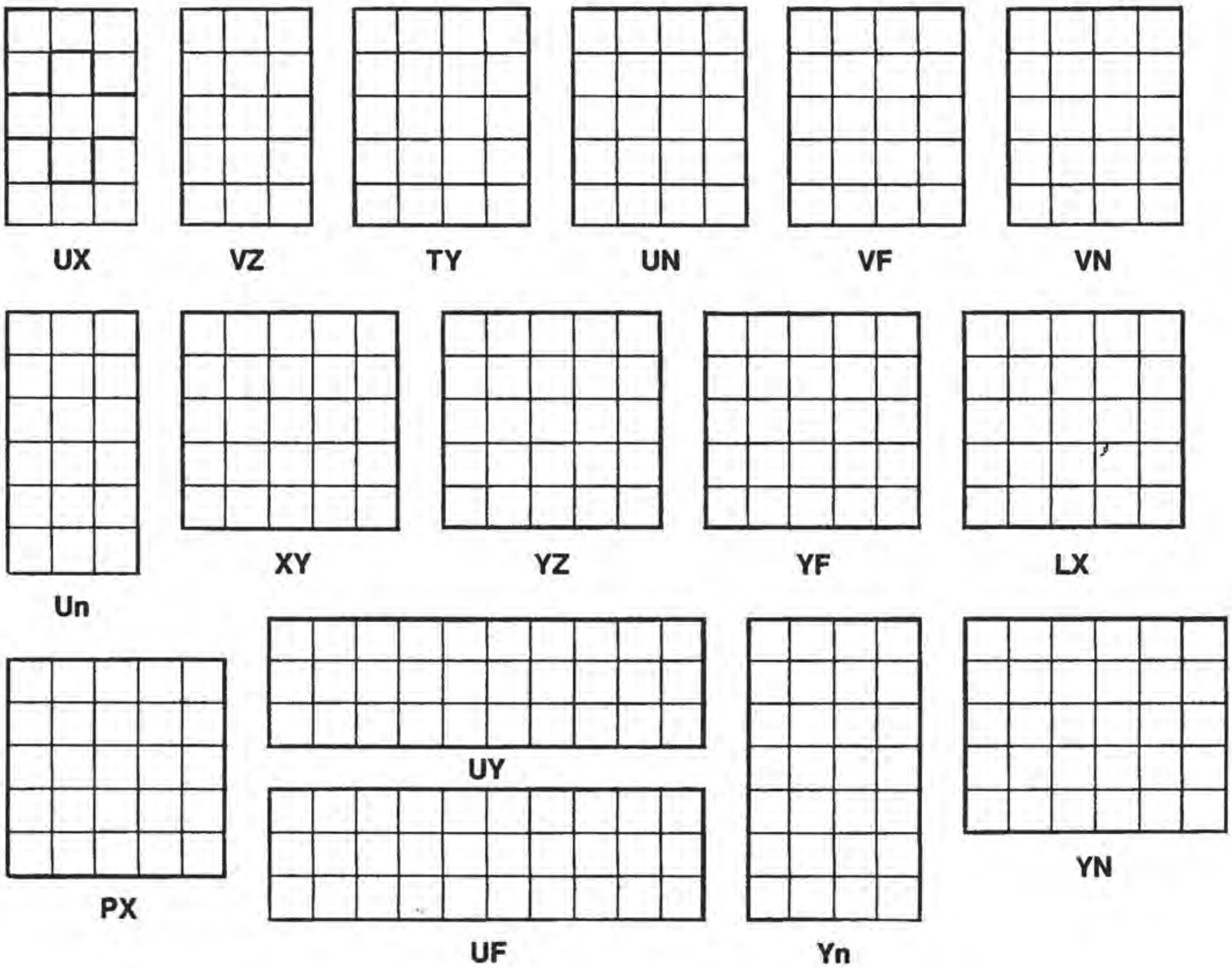
For each of the polyominoes listed below, find the smallest rectangle that it tiles. Show your solutions on the grid.

1. Tetrominoes: l, n, t ★
2. Pentominoes: L, P, Y ★



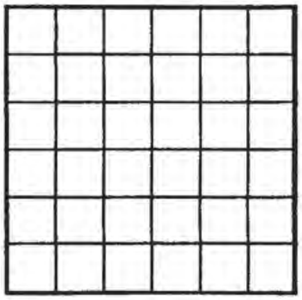
Tiling with Polyomino Pairs

All these squares and rectangles can be tiled by pairs of polyominoes. For example, you can tile the 3-by-5 rectangle with two U's and one X. Show how to cover each shape below using the given polyominoes.

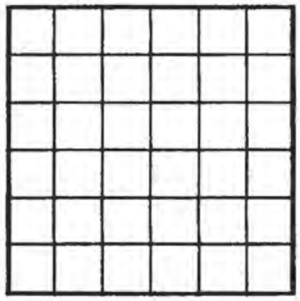


More Tiling with Polyominos

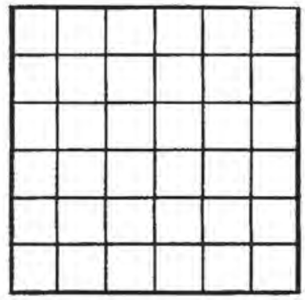
Now tile these squares and rectangles with pairs of polyominoes. Show how you covered each shape by using the given polyominoes.



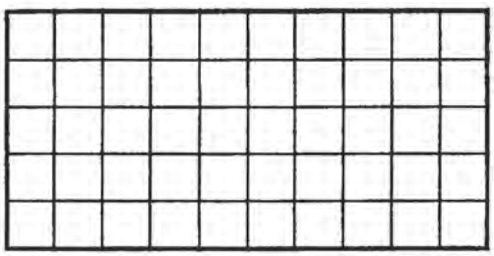
Tn



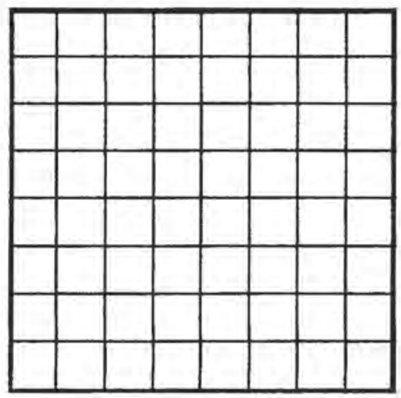
Yn



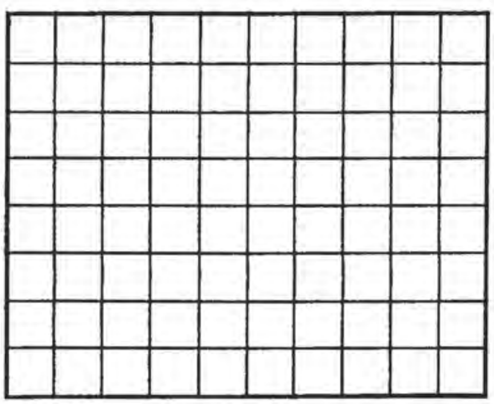
Pn



TN



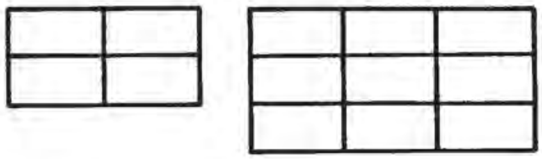
Ln



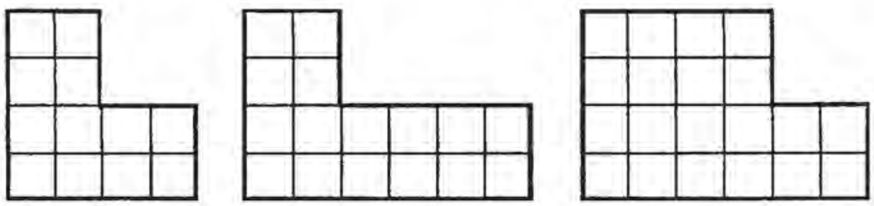
TW

Rep-tiles

A shape is a rep-tile if it can be used to tile a larger copy of itself. For example, a domino is a 1-by-2 rectangle. If you double its dimensions, you get a 2-by-4 rectangle. If you triple them, you get a 3-by-6 rectangle. These larger copies of the domino can be tiled by dominoes. Therefore the domino is a rep-tile.



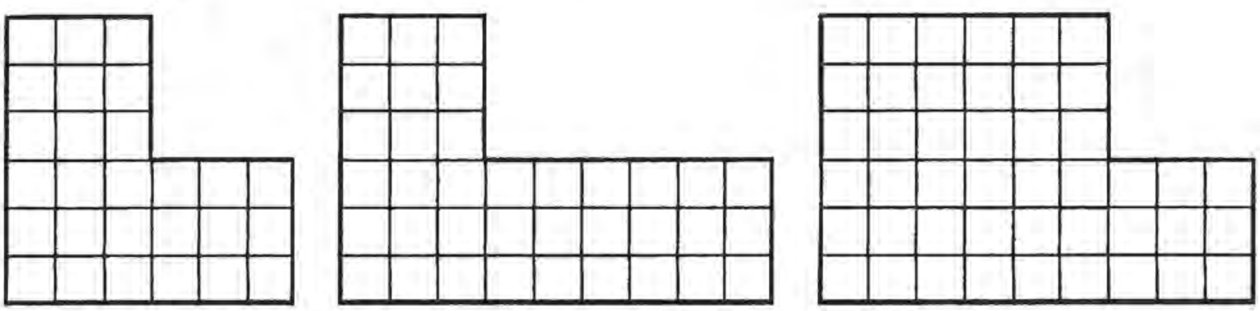
1. The bent triomino, the l tetromino, and the P pentomino are rep-tiles. Here are these polyominoes with their dimensions doubled. Tile the blown-up version of each one with the original shape.



2. How many original shapes did you need to cover each doubled figure?_______

3. When the dimensions of a figure are doubled, its area is multiplied by________.

4. Here are the bent triomino, the l tetromino, and the P pentomino with their dimensions tripled. Tile the blown-up version of each one with the original shape.

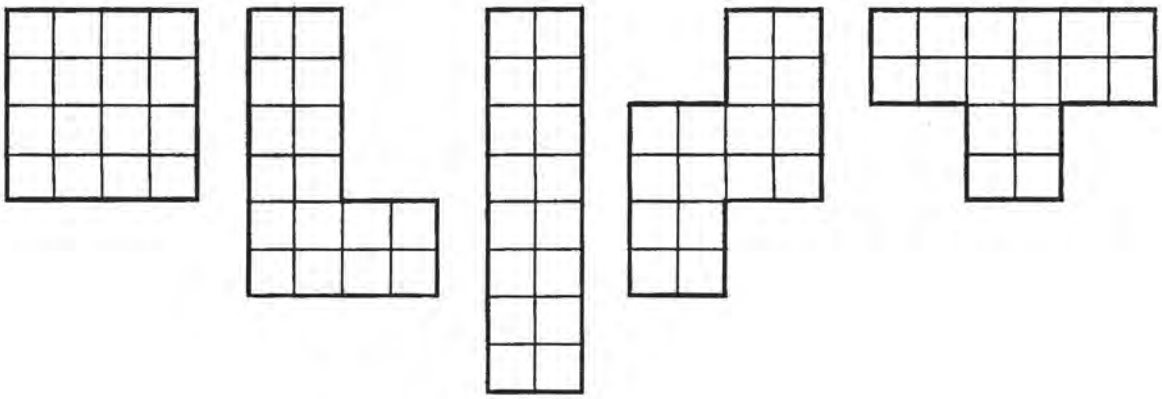


5. How many original shapes did you need to cover each tripled figure?________

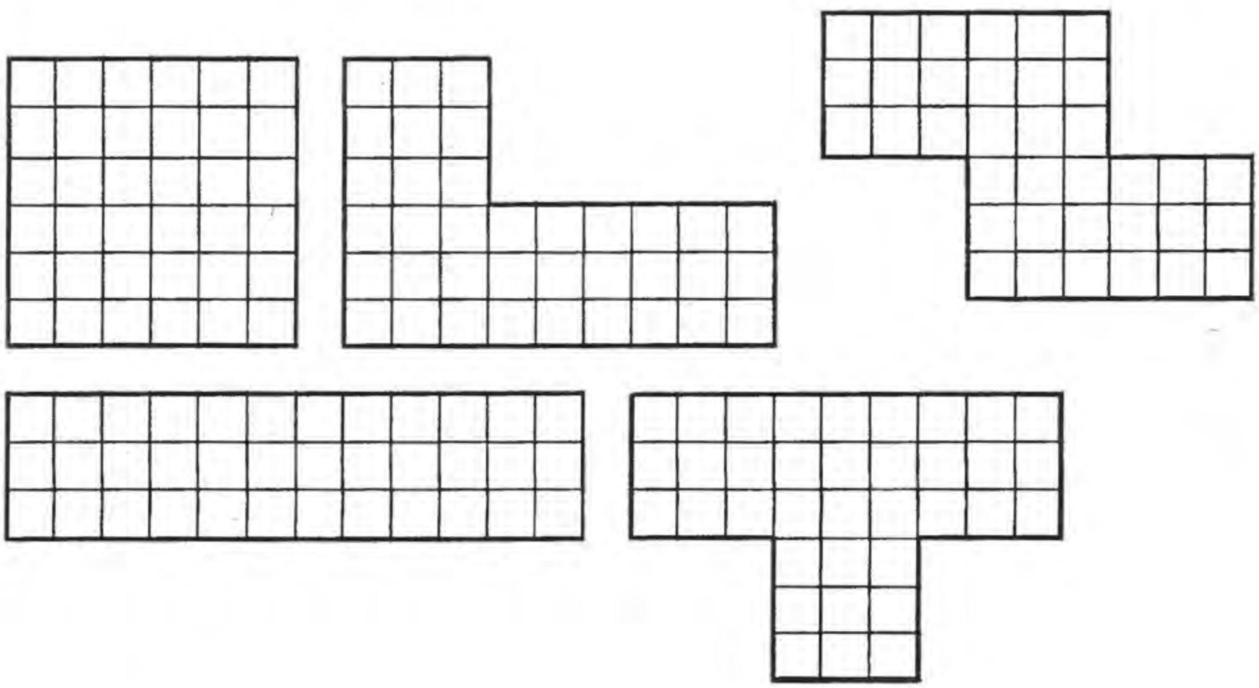
6. When the dimensions of a figure are tripled, its area is multiplied by ________ .

Doubled and Tripled Tetrominoes

1. Tile this set of doubled tetrominoes using only the I tetromino.

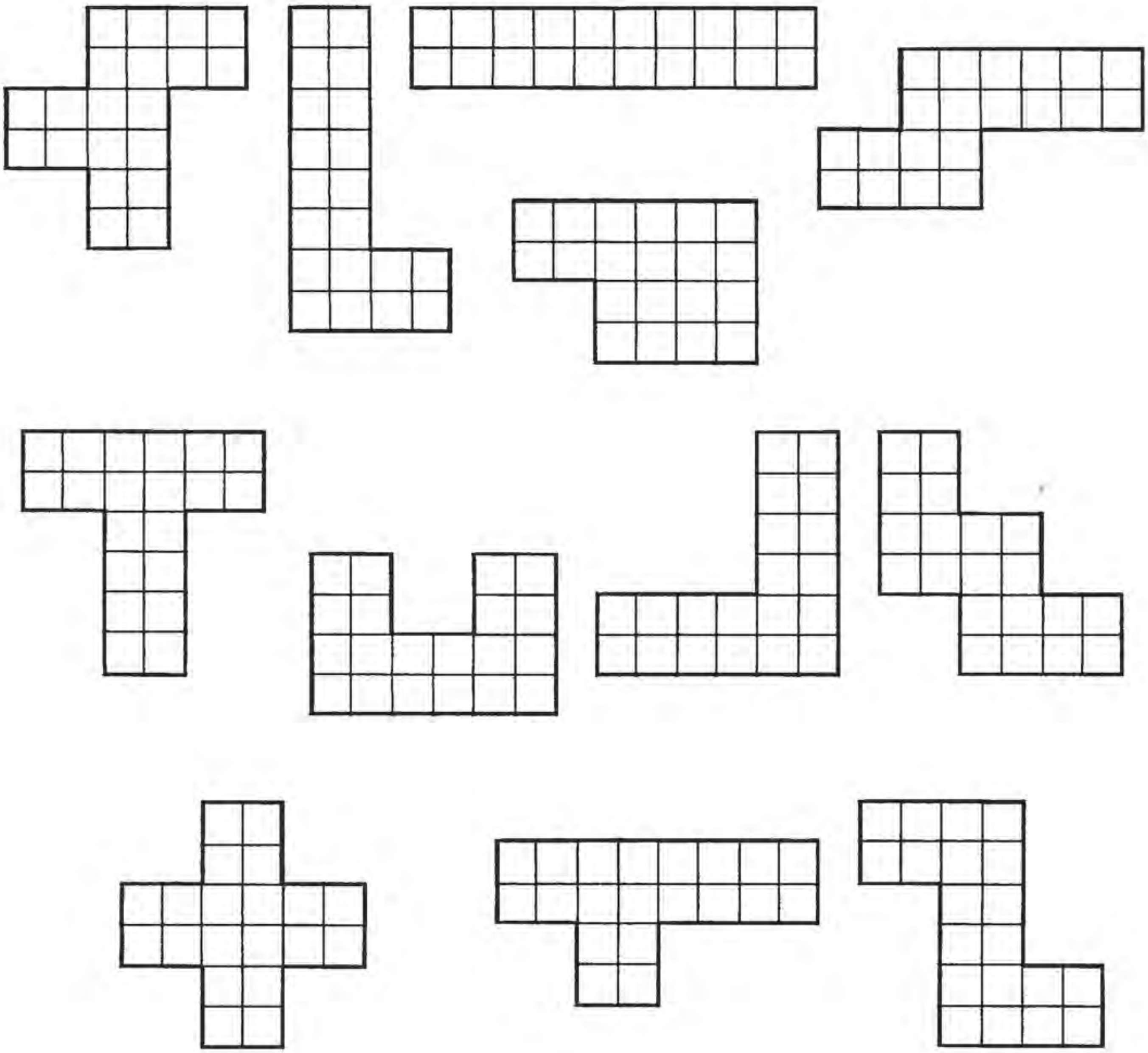


2. Now tile this set of tripled tetrominoes using just the I and t tetrominoes.



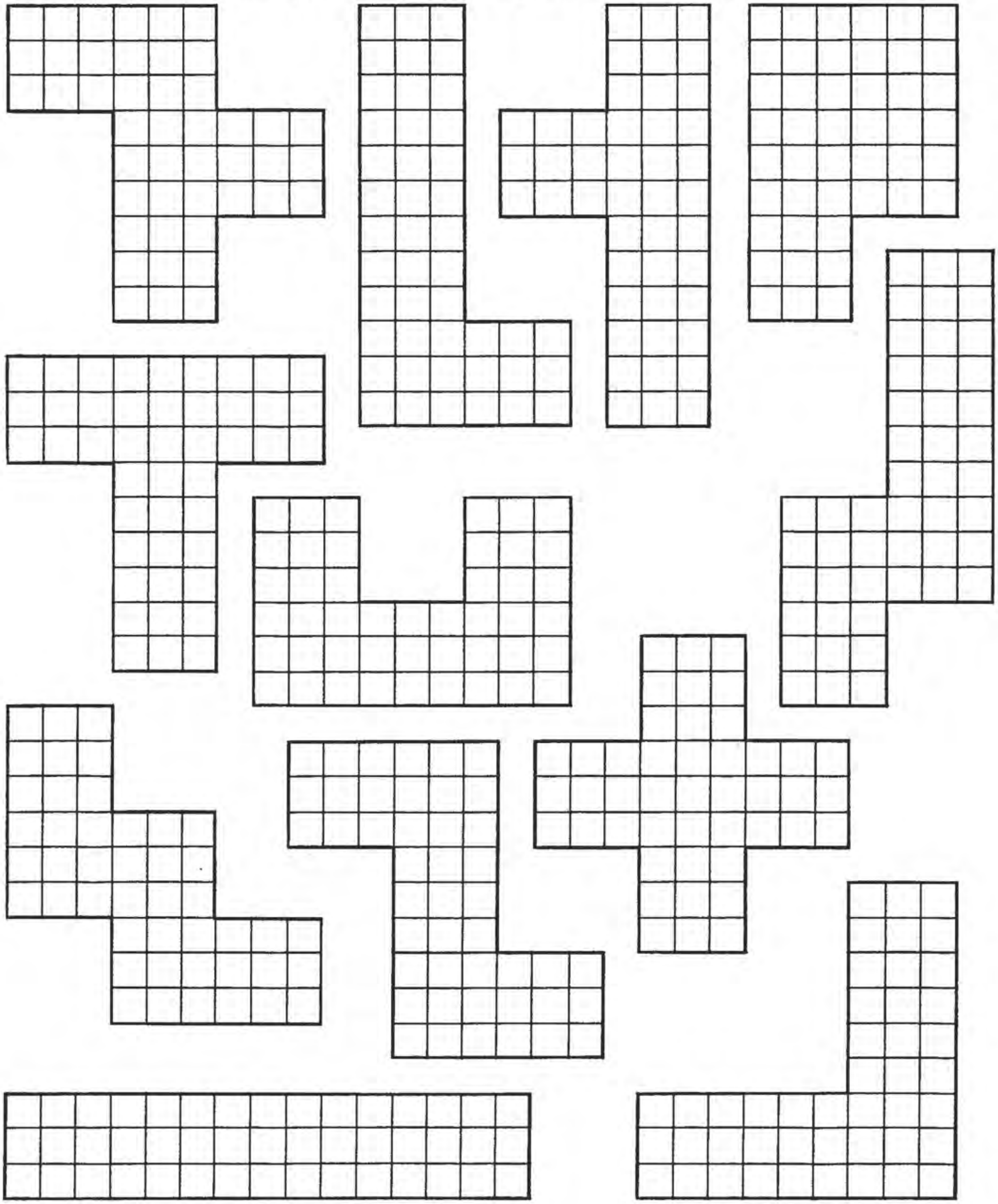
Doubled Pentominoes

Tile this set of doubled pentominoes using only the P and N pentominoes. (You need only three N's.) ★



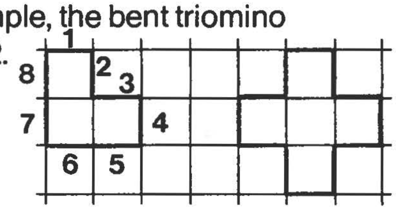
Tripled Pentominoes

Tile this set of tripled pentominoes using only the P and L pentominoes.

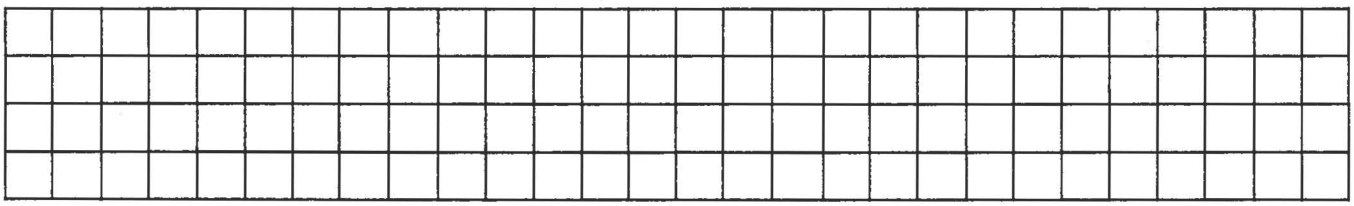


Perimeter 10

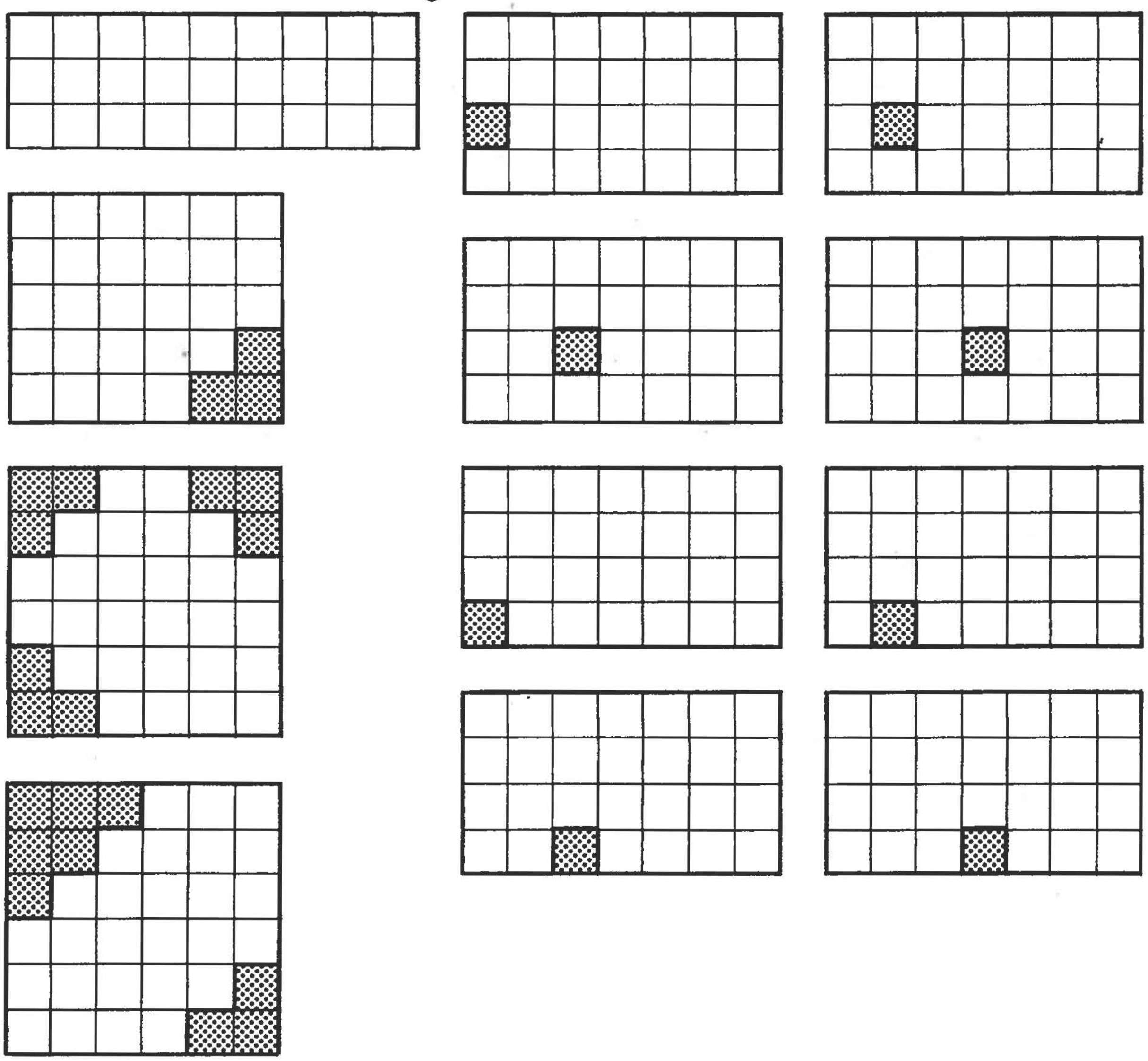
The perimeter of a polyomino is the distance around it. For example, the bent triomino has a perimeter of 8 and the X pentomino has a perimeter of 12.



1. Find all the polyominoes that have a perimeter of 10. List them here. ________ ★



2. Use the above polyominoes to cover these shapes. Do not use any polyomino more than once in each figure.



Perimeter and Area Table

The area of a polyomino is the number of square units it contains. For example, pentominoes have an area of 5.

1. Can you draw a polyomino with a perimeter that is an odd number?________
2. Experiment on grid paper. Fill out as much of this table as you can. It has been started for you. Look for patterns.

	PERIMETER	
AREA	*Shortest*	*Longest*
1	4	4
2	____	____
3	____	____
4	8	10
5	____	____
6	____	____
7	____	____
8	____	____
9	____	____
10	____	____
11	____	____
12	____	____
13	____	____
14	____	____
15	____	____
16	____	____
17	____	____
18	____	____
19	____	____
20	____	____
21	____	____
22	____	____
23	____	____
24	____	____

Perimeter-Area Predictions

1. Draw polyominoes each having an area of 10 that have all the possible perimeters you can find. Use the grid below and more grid paper if you need it. ★
2. Draw polyominoes each having an area of 16 with all possible perimeters. ★
3. Predict the longest possible perimeters for shapes with these areas. Experiment on grid paper to test your predictions.

   36 ________ 40 ________ 100 ________ 99 ________ 101 ________
4. Can you state a method or formula to get the longest perimeter when you know the area? ★ ____

   ____
5. Predict the shortest possible perimeters for shapes with these areas. Experiment on grid paper.

   36 ________ 40 ________ 100 ________ 99 ________ 101 ________

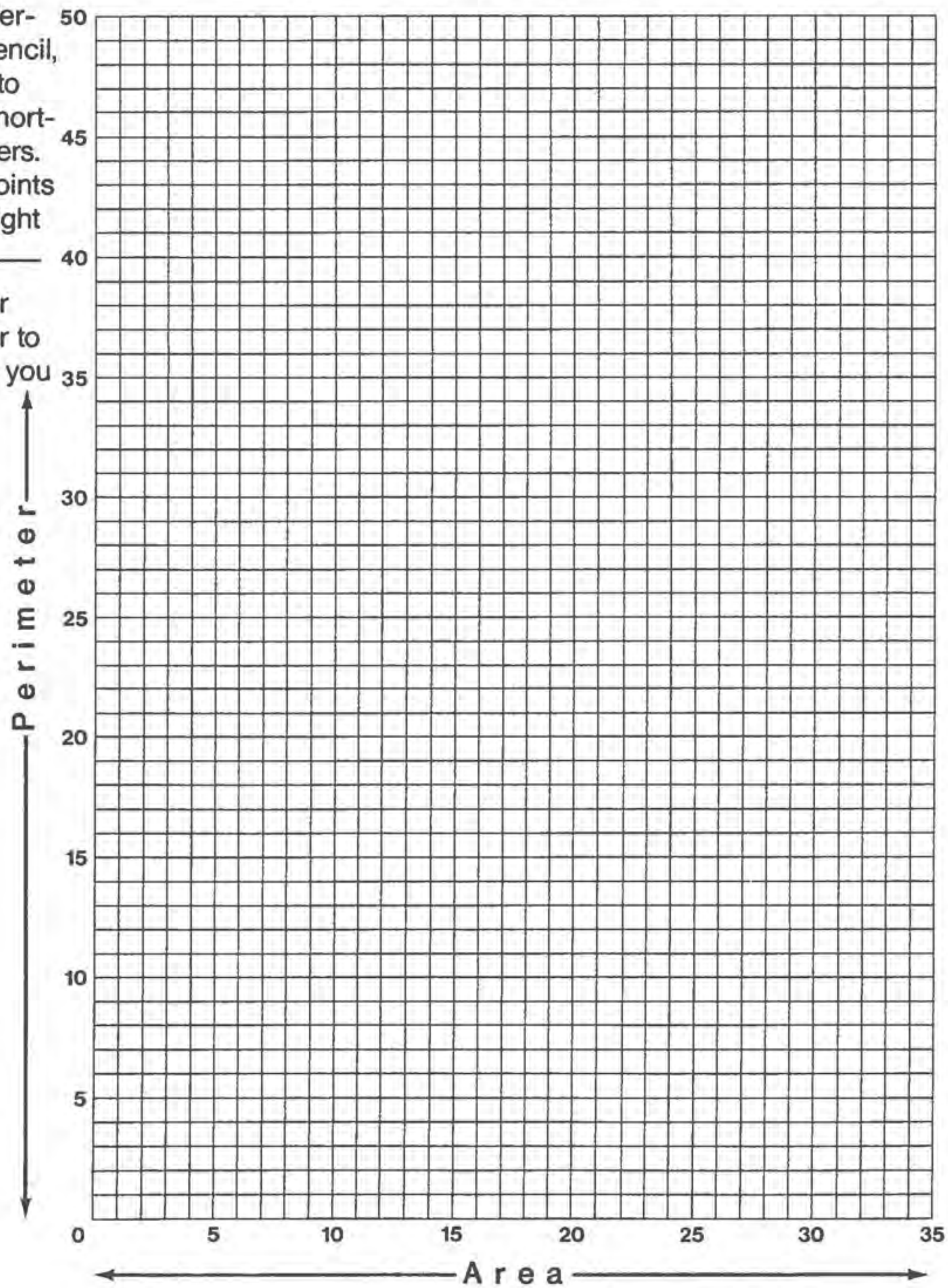
Perimeter-Area Graphing

1. Look at your table showing perimeter and area (page 35). Put the information about the longest perimeters on this graph. Since the longest perimeter for an area of 4 is 10, put a dot at point (4,10) on the graph. (Start at 0, go 4 spaces over to the right and 10 spaces up.) What do you notice about all these points?

____

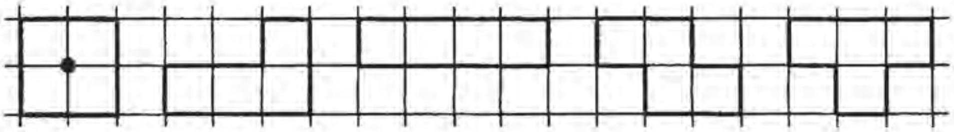
2. Using a different color pencil, make dots to show the shortest perimeters. Do these points lie in a straight line? ________

3. Extend your graph as far to the right as you can.



Eyes

Let's call the points of intersection of the grid lines inside a polyomino *eyes*. The square tetromino has 1 eye. No other tetrominoes have any eyes.



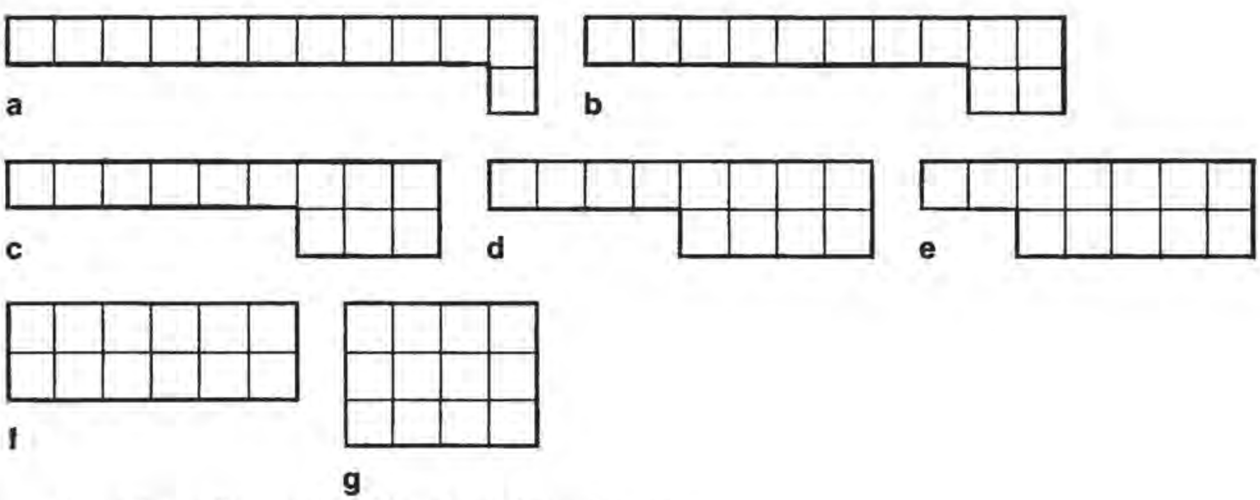
1. Look at the tetrominoes and fill out this table. It has been started for you.

Tetromino:	square	l	i	n	t
Eyes:	1	0	0	0	0
Perimeter:	___	___	___	___	___

2. Now fill out this table about pentominoes. You may need to use grid paper and draw the pentominoes. ★

Pentomino:	F	L	I	P	N	T	U	V	W	X	Y	Z
Eyes:	___	___	___	___	___	___	___	___	___	___	___	___
Perimeter:	___	___	___	___	___	___	___	___	___	___	___	___

3. Draw in all the eyes in the 7 figures below.



4. Then fill out this table about those figures.

Figure:	a	b	c	d	e	f	g
Area:	___	___	___	___	___	___	___
Eyes:	___	___	___	___	___	___	___
Perimeter:	___	___	___	___	___	___	___

5. Think of figures that have the same area. As the number of eyes increases, does the perimeter get longer or shorter? ________

Perimeter-Area Formulas

Here is how to find the longest perimeter for a given area. Call p the perimeter and a the area.

$$p = (2 \times a) + 2$$

or

$$p = (a + 1) \times 2$$

In other words, you double the area, then add 2, or add 1 to the area, then double it.

1. Check to see that this works for polyominoes that have an area of 5 or less. Check it for areas of 10, 12, and 16.
2. If you keep the area the same, but increase the number of eyes by 1, what happens to the perimeter?______________________________

____

Here is how to find the perimeter of a polyomino. Call a the area and e the number of eyes.

$$p = (2 \times a) + 2 - (2 - e)$$

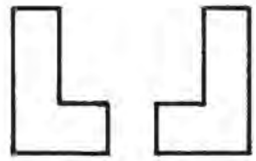
or

$$p = (a + 1 - e) \times 2$$

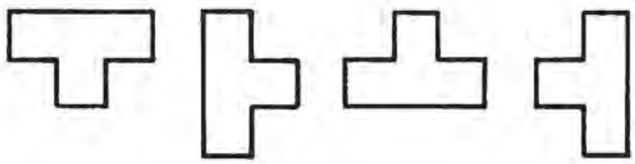
3. Check to see that this works for tetrominoes, pentominoes, and the 12-ominoes shown on page 38. Use the space below for your figuring.

One-Sided Polyominoes

If polyominoes were one-sided and could not be flipped over, there would have to be two l tetrominoes. You could call them the *right* l and the *left* l.

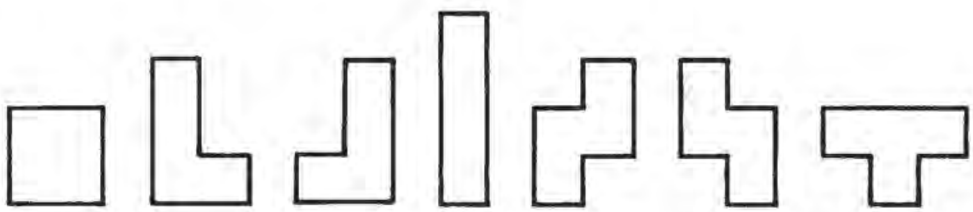


There would still be only one t tetromino. You can slide it into any position without flipping it over.

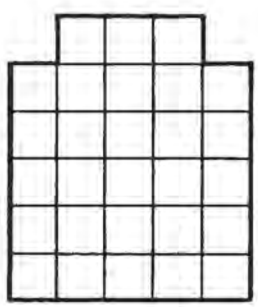


1. There would need to be two of one other tetromino. Which one is it? ________

Here are all seven *one-sided tetrominoes.*

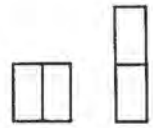


2. Use these seven shapes to cover the figure below. Do not use any shape more than once.



3. Find the one-sided pentominoes. Draw them on grid paper. ★

Polyrectangles

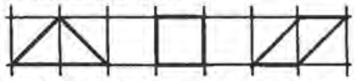


You can make polyrectangles on rectangle paper. There are 2 directangles.

1. Find all the trirectangles. Show them below. ★
2. Find all the tetrarectangles. ★
3. Find all the pentarectangles. You may need to use some of the rectangular grid paper in the back of this book. ★

Polytans

This is an isosceles right triangle. It is half of a square. Five of the seven pieces of the old Chinese *tangram* puzzle are triangles like this. Let's call the figures made by combining them *polytans*. Here are the ditans below.



1. Find all the tritans. Show them on the grid below. ★
2. Find all the tetratans. Draw them too. Sometimes they are called *supertangrams*. ★

HINTS

Tetrominoes, page 4
There are 5 distinct tetrominoes.

Pentominoes, page 5
There are 12 distinct pentominoes.

Polyomino Names, page 6
Eight of them can be folded into a box. If you have trouble finding them, experiment with one-inch grid paper and scissors.

Making Polyomino Rectangles, page 8

1. There are 25 rectangles. Their sizes are:

2 x 2	2 x 3	2 x 4	3 x 3
2 x 5	2 x 6	3 x 4	2 x 7
3 x 5	4 x 4	2 x 8	3 x 6
2 x 9	2 x 10	4 x 5	3 x 7
2 x 11	2 x 12	3 x 8	4 x 6
5 x 5	2 x 13	3 x 9	2 x 14 and 4 x 7

2. Use the monomino in the 3-by-9 rectangle.

Pentomino Family Relationships, page 11

1. Look at your tetromino family trees.
2. Students may find it helpful to draw a full family tree for each of the pentominoes.

Hexomino Envelopes, page 15
There are 6 hexomino envelopes.

Classifying the Hexominoes, page 16
Eleven can be folded into cubes. If you have trouble finding them, use one-inch grid paper and scissors to experiment.

Tiling Rectangles, page 27

1. One of them does not tile any rectangle.
2. The Y pentomino tiles a 5-by-10 rectangle.

Doubled Pentominoes, page 32
Use one N in each of these doubles: N, V, W.

Perimeter 10, page 34

1. There are 6 of them.

Perimeter-Area Predictions, page 36

1. There are 5 different shapes possible.
2. There are 10 different shapes possible.
4. Look at the table on page 35. Double each area number and then look at the corresponding longest perimeter. Find a pattern.

Eyes, page 38

2. Only 1 pentomino has an eye.

One-Sided Polyominoes, page 40

3. There are 18 of them.

Polyrectangles, page 41

1. There are 3 of them.
2. There are 9.
3. There are 21.

Polytans, page 42

1. There are 4 of them.
2. There are 14.

SOLUTIONS

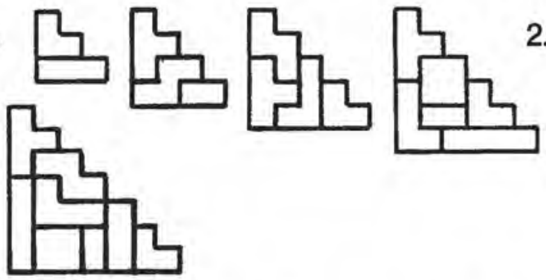
Tetrominoes, page 4,
See *Polyomino Names*, page 6.

Pentominoes, page 5
See *Polyomino Names*, page 6.

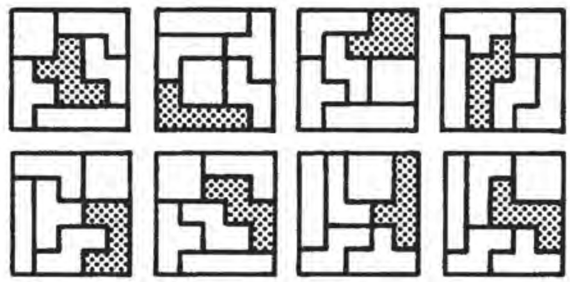
Polyomino Names, page 6
F, L, N, T, W, X, Y, Z

Some Polyomino Puzzles, page 7

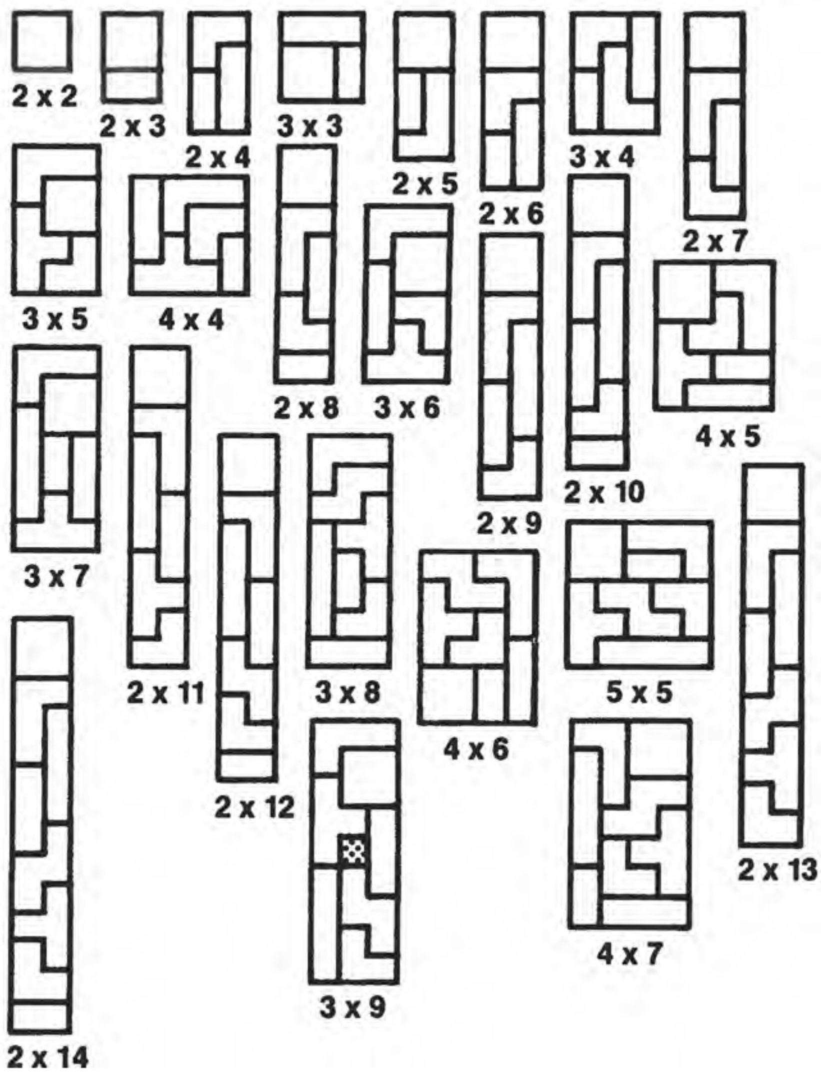
1.



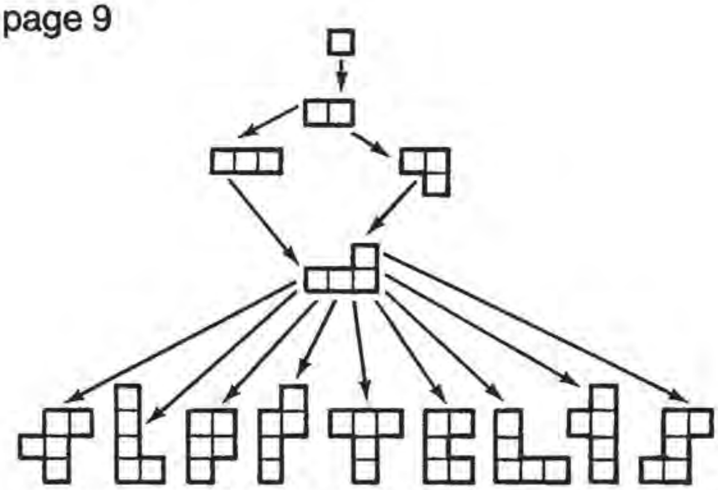
2.



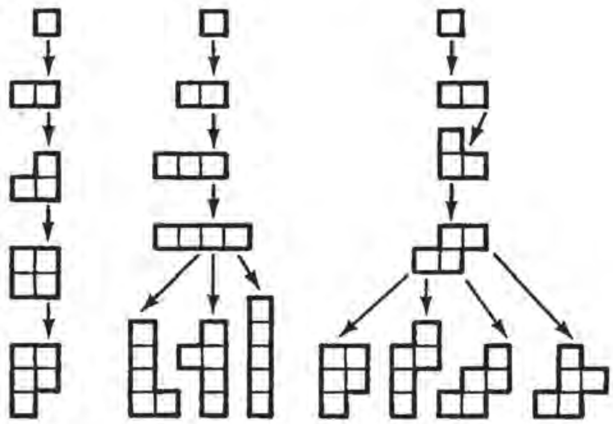
Making Polyomino Rectangles, page 8



Family Trees, page 9



Family Trees, page 10



Pentomino Family Relationships, page 11

1. The P has 4 parents.
2. L and Y
3. F, N, P
4. only the W
5. I and W
6. The L has 11 children.
7. The X has only 2 children.

Envelopes, page 12

1. 1 x 4   i
2. 2 x 2   square
3. 3 x 2   n, l, t

Pentomino Envelopes, page 13

1. 1 x 5   I
2. 2 x 3   P, U
3. 2 x 4   L, N, Y
4. 3 x 3   F, T, V, W, X, Z

Hexominoes, page 14

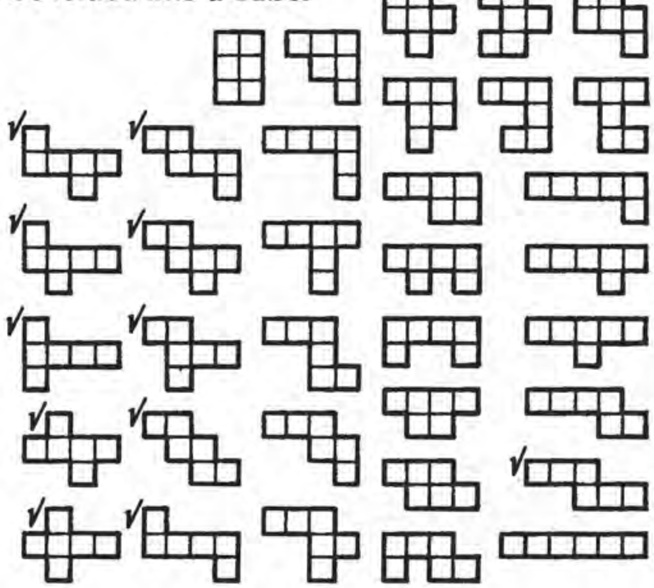
There are 35 hexominoes.

Hexomino Envelopes, page 15

~~2 by 2~~	~~1 by 5~~	2 by 3	2 by 5
3 by 3	~~3 by 5~~	2 by 4	1 by 6
3 by 4	~~2 by 6~~	~~4 by 4~~	~~3 by 6~~

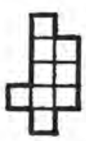
Classifying the Hexominoes, page 16

√ = Can be folded into a cube.



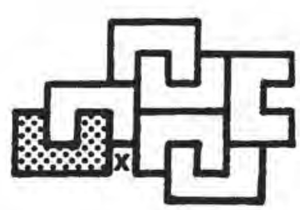
Minimum Covers, page 17

1. **2.** **3.**

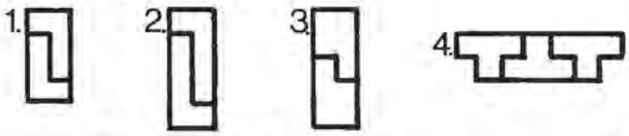
Areas:  6  9  12

Tiling with Other Polyominoes, page 19

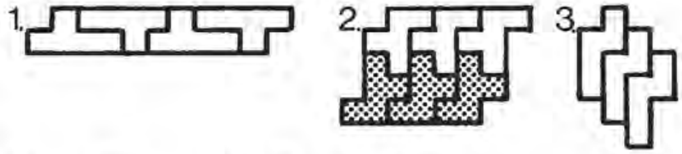
1. yes-Extend the horizontal stripe in both directions, and copy it above and below.
2. no-The only way to fill the space inside the U on the left is to add the shaded U. But this creates a space — x — that can't be filled by a U.



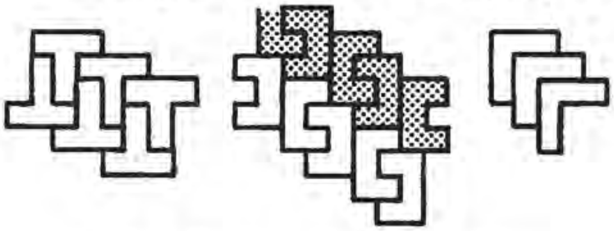
Tiling, page 20



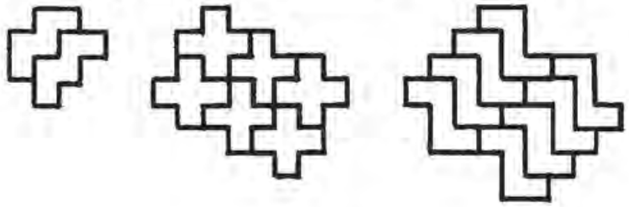
Tiling with Pentominoes, page 21



Tiling with Pentominoes, page 22

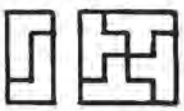


More Tiling with Pentominoes, page 23



Tiling Rectangles, page 27

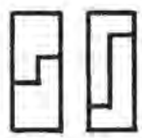
1.



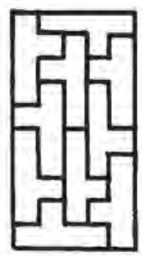
The n tetromino doesn't tile.

2 by 4 **4 by 4**

2.

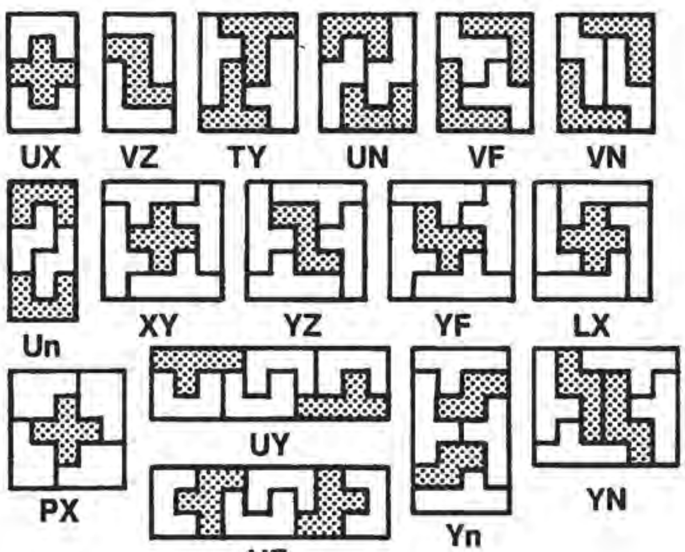


2 by 5 **2 by 5**

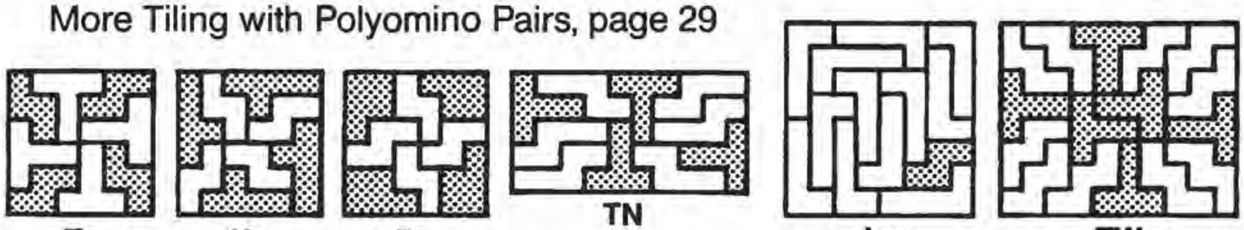


5 by 10

Tiling with Polyomino Pairs, page 28

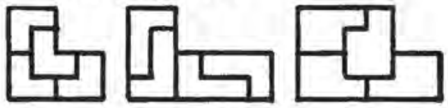


More Tiling with Polyomino Pairs, page 29



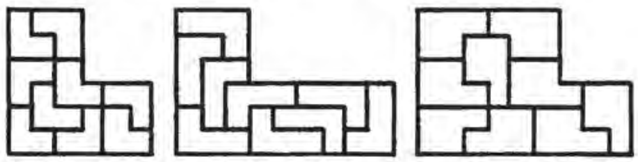
Tn **Yn** **Pn** **TN** **Ln** **TW**

Rep-tiles, page 30

1. 

2. 4

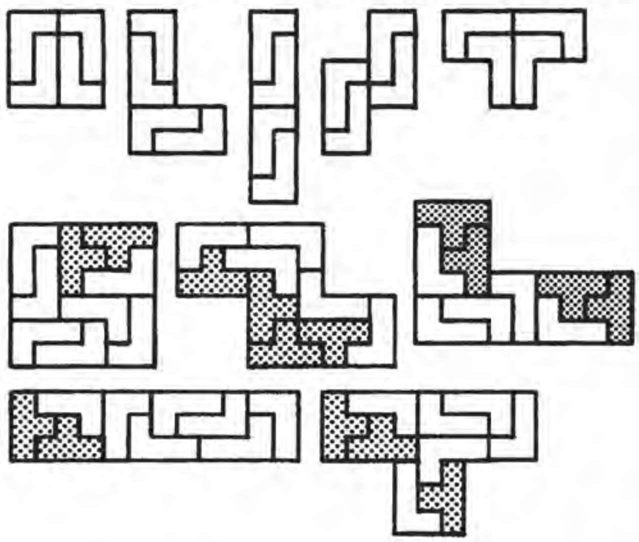
3. 4

4. 

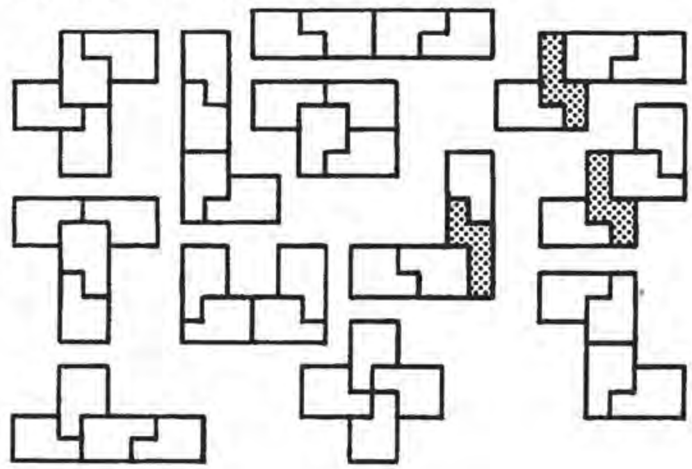
5. 9

6. 9

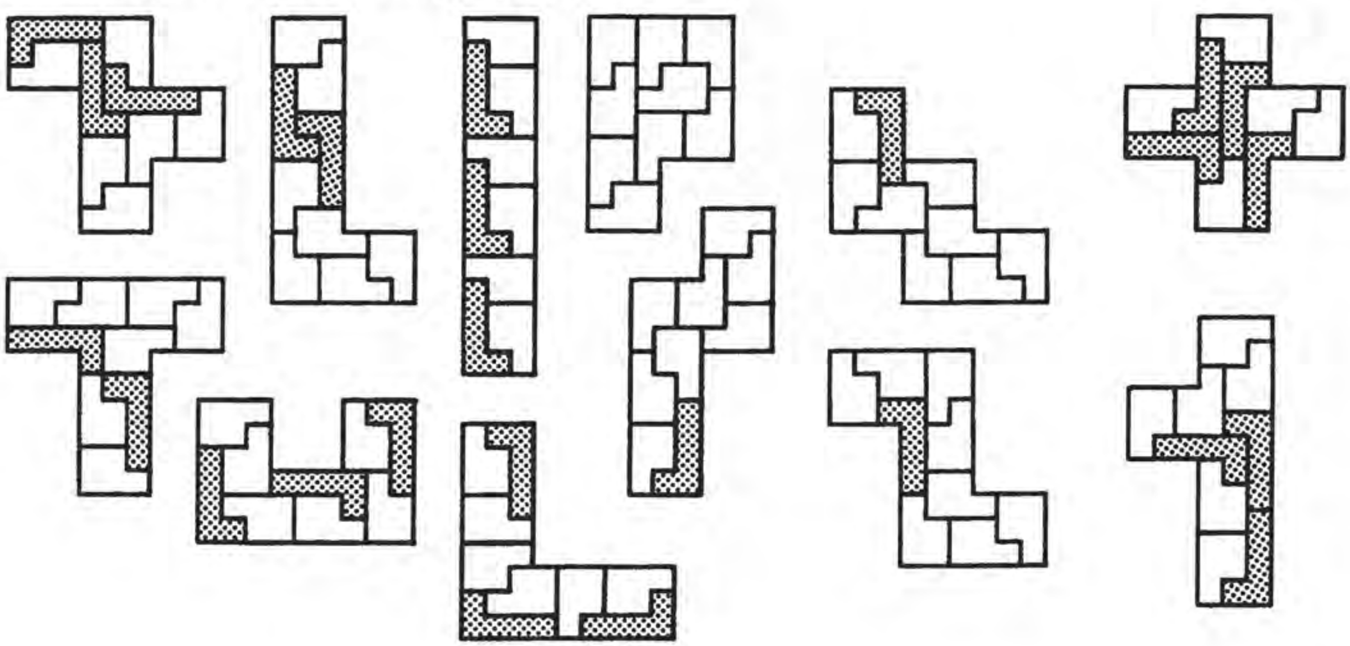
Doubled and Tripled Tetrominoes, page 31



Doubled Pentominoes, page 32

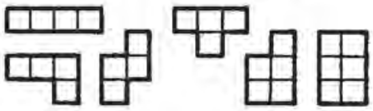


Tripled Pentominoes, page 33

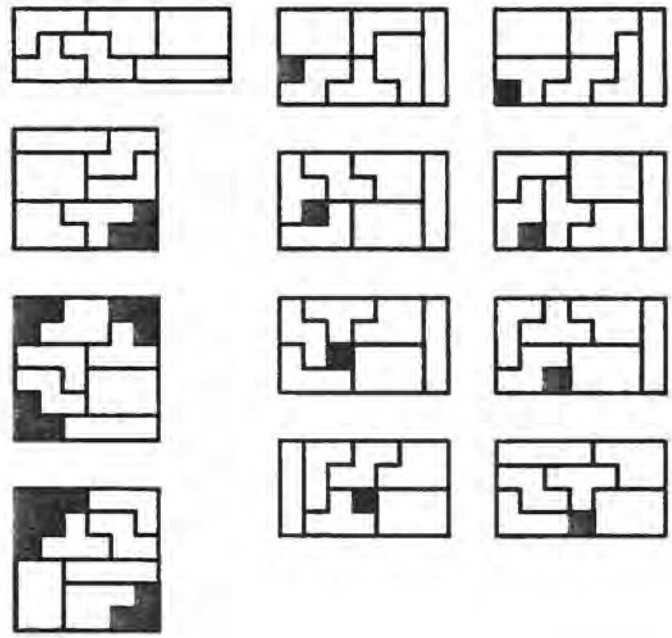


Perimeter 10, page 34

1.



2.



Perimeter and Area Table, page 35

1. no

2.

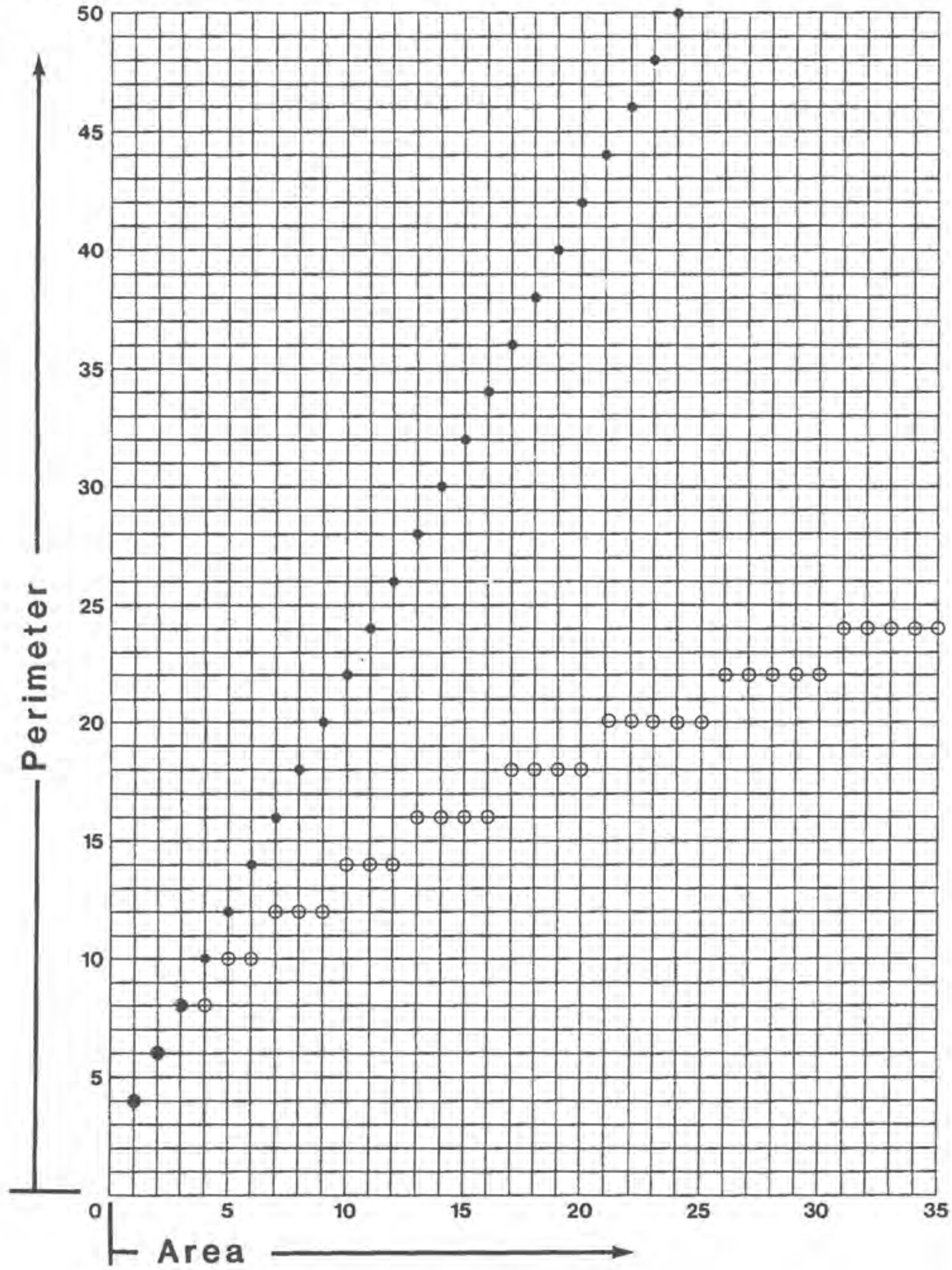
AREA	PERIMETER Shortest	PERIMETER Longest
1	4	4
2	6	6
3	8	8
4	8	10
5	10	12
6	10	14
7	12	16
8	12	18
9	12	20
10	14	22
11	14	24
12	14	26
13	16	28
14	16	30
15	16	32
16	16	34
17	18	36
18	18	38
19	18	40
20	18	42
21	20	44
22	20	46
23	20	48
24	20	50

Perimeter-Area Predictions, page 36

3. 74 82 202 200 204
4. See *Perimeter-Area Formulas* on page 38.
5. 24 26 40 40 42

Perimeter-Area Graphing, page 37

1. They lie in a straight line.
2. no



Eyes, page 38

1.

Tetromino:	square	l	i	n	t
Eyes:	1	0	0	0	0
Perimeter:	8	10	10	10	10

2. All have 0 eyes and perimeters of 12, except the P, which has 1 eye and a perimeter of 10.

4.

Figure:	a	b	c	d	e	f	g
Area:	12	12	12	12	12	12	12
Eyes:	0	1	2	3	4	5	6
Perimeter:	26	24	22	20	18	16	14

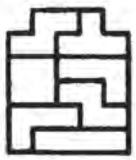
5. shorter

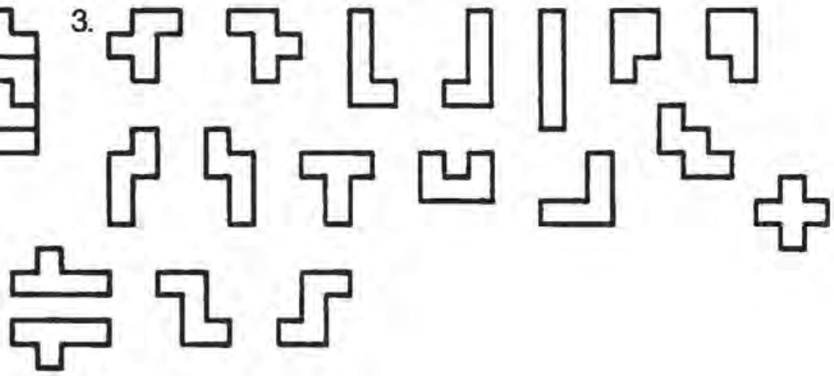
Perimeter-Area Formulas, page 39

2. It decreases by 2.

One-Sided Polyominoes, page 40

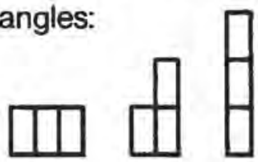
1. the n

2. 

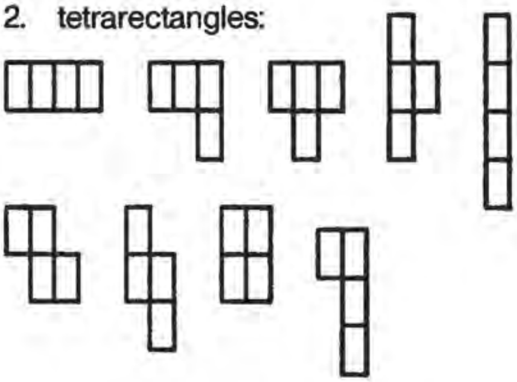
3. 

Polyrectangles, page 41

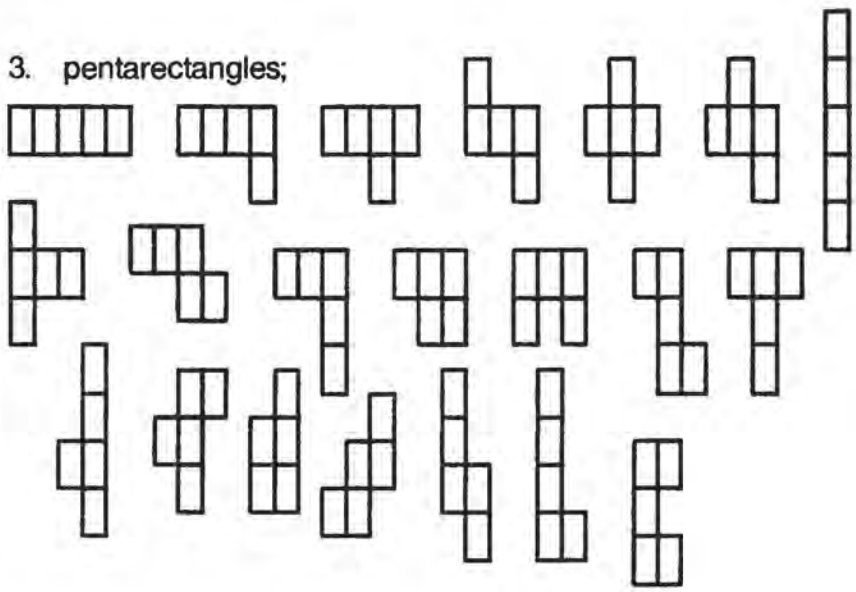
1. trirectangles:



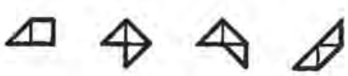
2. tetrarectangles:

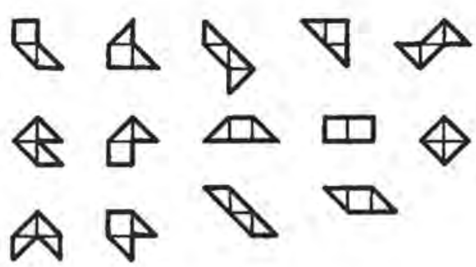


3. pentarectangles;



Polytans, page 42

1. 

2. 

COMMENTS FOR THE TEACHER

Finding the Polyominoes, page 3

If possible, this whole chapter should be taught before showing any plastic polyominoes to the students.

You may use this opportunity to discuss the prefixes di-, tri-, tetr-, pent-, hex-, and hept-. They will be used many times in this book.

In addition to using grid paper to look for the pentominoes, your students can use orange pattern blocks (squares), geo-boards, one-inch cubes, or interlocking cubes. However, remind students to seek only the "flat" combinations.

Polyomino Names, page 6

Your students may want to make up their own names for the polyominoes. However, they will need to know the names used in this book.

Making Polyomino Rectangles, page 8

Students can solve these puzzles directly on quarter-inch grid paper, using a pencil and good eraser. It would be easier to work on one-inch grid paper, using cardboard polyominoes made from the patterns in the back of this book.

Pentomino Family Relationships, page 11

You might use this opportunity to discuss the meanings, in real life, of such obscure ideas as "second cousin once removed," etc.

Envelopes, page 12

This section is actually a guided hunt for the hexominoes. If you wish, you can let your students look for the hexominoes with no guidance, just by using the page titled *Hexominoes*. However, the systematic approach helps settle these uncertainties: Have we found them all? Are there any duplicates? These questions are easier to answer when the hexominoes are classified by envelopes.

Many pentomino puzzles and problems can be adapted for the hexominoes.

Minimum Covers, page 17

It is not necessary for your students to find the covers shown in the solutions. Let them find what they can, then try to beat that record.

Tiling, page 18

Instead of working on grid paper, your students could work on oversized unlined paper and trace plastic polyominoes repeatedly.

Encourage students to color their tilings of the plane for a bulletin board display of the most spectacular drawings.

Rep-tiles, page 30

You and your students may investigate other shapes to see if they are rep-tiles. For example, all but one of the pattern blocks are rep-tiles.

Perimeter 10, page 34

Your students may work right on this puzzle page with pencil and eraser. However, it is easier to work on one-inch grid paper, with pieces made from the patterns in the back of this book.

You may find all shapes that have a perimeter of 12 (there are 25) and create puzzles for them.

Perimeter and Area Table, page 35

Some students find it hard to accept that it is impossible to have a polyomino with an odd-numbered perimeter. The reason it is impossible is that each side, or part of a side, of a polyomino can be matched with another side, or part of a side. It has an "opposite." Therefore, the perimeter must be an even number.

The pattern for the maximum perimeter becomes evident after a time. Try to get students to verbalize it. If they are mathematically mature, see if they can create a formula.

The pattern for the minimum perimeter is much harder to see. One way to state it is that it remains constant for the sequences of one, one, two, two, three, three, etc. consecutive values of the area. Then it increases by two.

Perimeter-Area Predictions, page 36

This is the beginning of the most quantitative part of the book so far. It is best suited to eighth-grade students or gifted students in the lower grades. You may want just to skip the rest of this chapter.

Perimeter-Area Graphing, page 37

You might have students graph not only maximum and minimum perimeters, but all possible perimeters. They would then fill up the entire space between minimum and maximum perimeter, though only on even, whole number values of the perimeter.

Perimeter-Area Formulas, page 39

Notice that the perimeter cannot be odd since it is obtained by doubling a number. The lowest perimeter can be found from the area like this: It is the smallest even number greater than four times the square root of the area.

One-Sided Polyominoes, page 40

Many problems and puzzles involving ordinary polyominoes can be investigated for one-sided polyominoes.

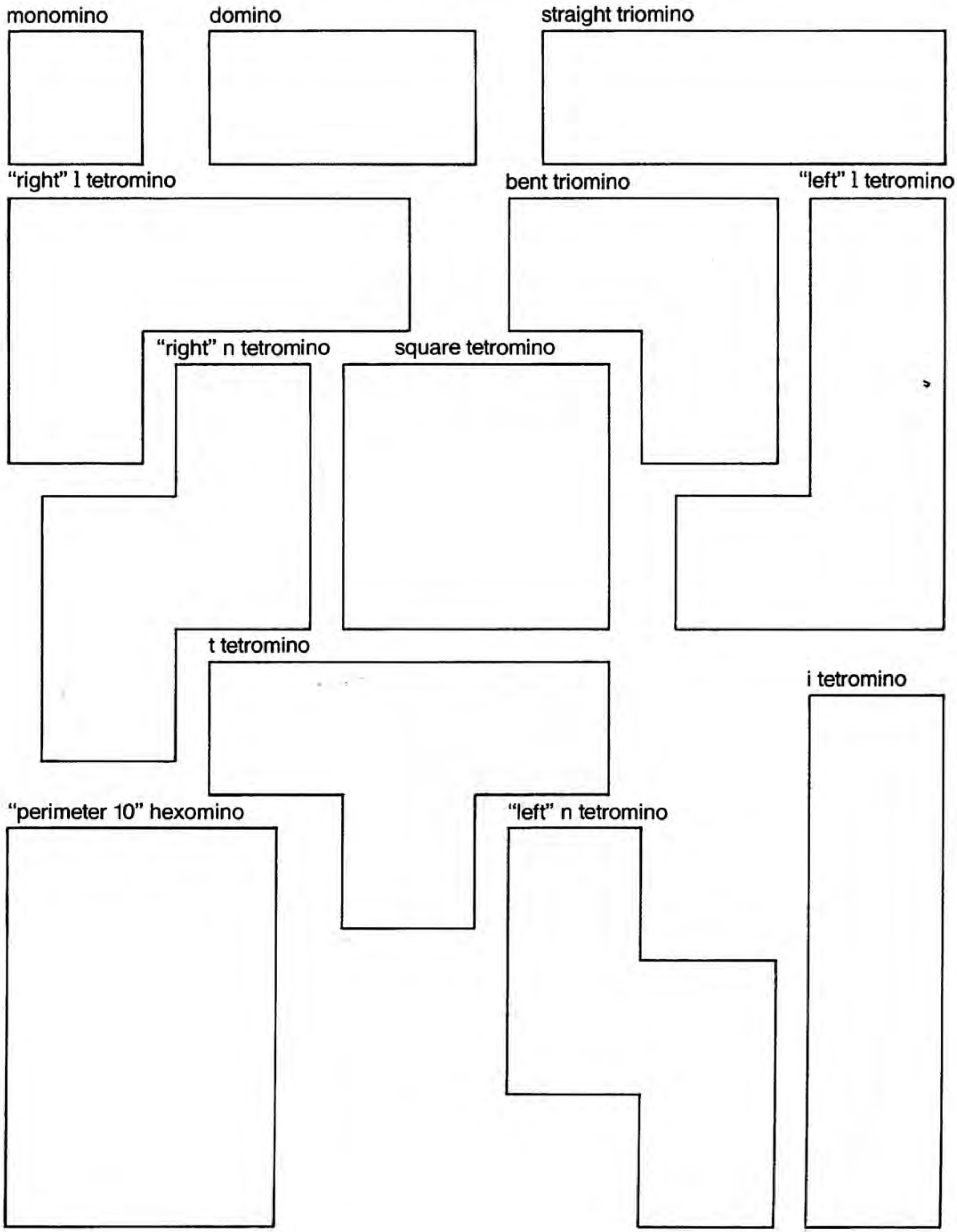
Polyrectangles, page 41

These shapes are closely related to the polyominoes. Each polyomino yields a "wide" and a "tall" version of itself among the polyrectangles, except for those that are symmetrical around a diagonal line (the square tetromino, the V, W, and X pentominoes). They yield only one polyrectangle each.

Polytans, page 42

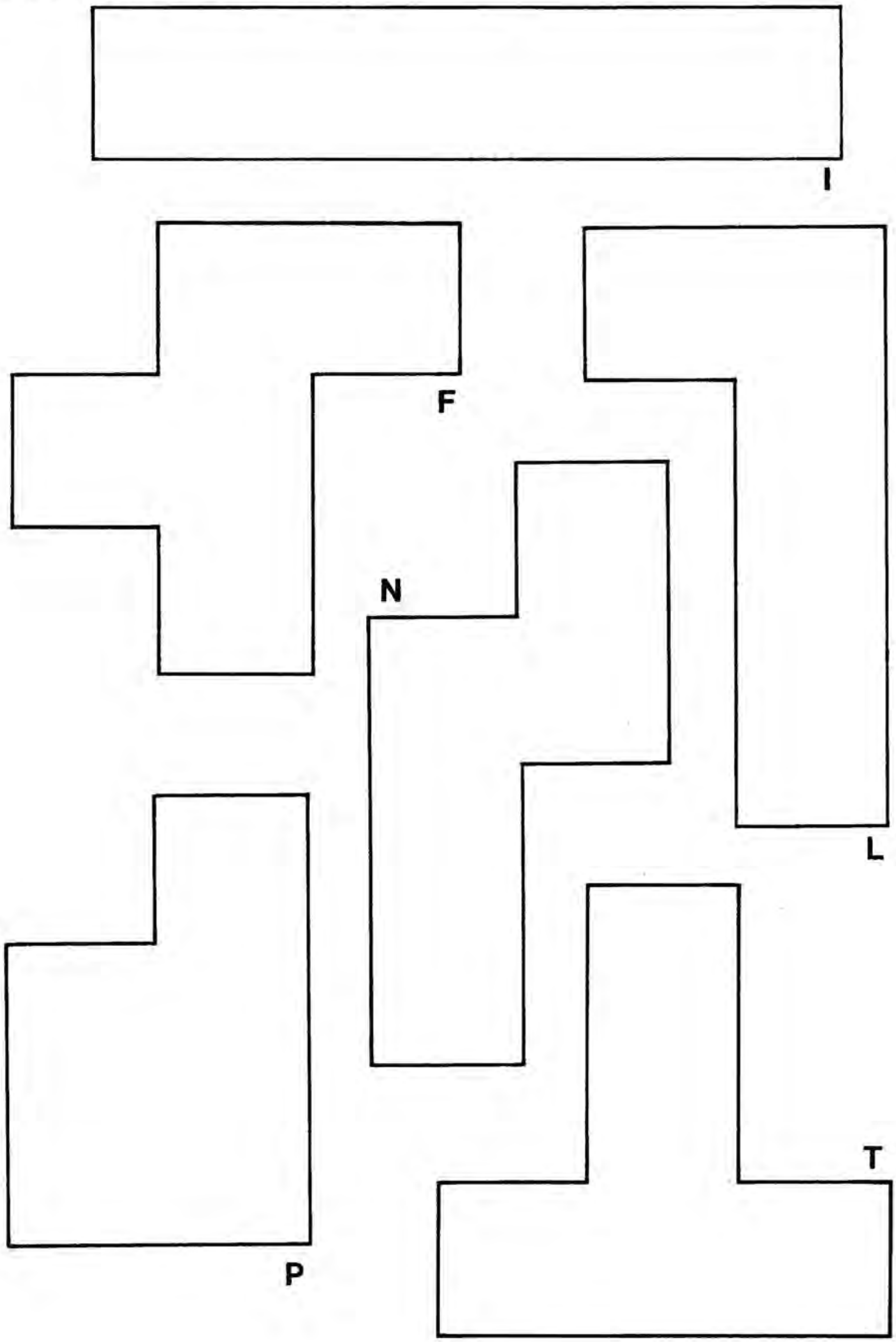
Use this opportunity to introduce tangram and Supertangram™ puzzles.

SELECTED POLYOMINO PATTERNS



PENTOMINO PATTERNS

Use these pages as patterns to make your own set of pentominoes or as a check to make sure you have a complete set.



PENTOMINO PATTERNS